Goals & Objectives

Number and Operations
Geometry and Measurement
Data Analysis and Probability
Algebra
Number and Operations
The learner will perform operations with numbers and expressions to solve problems.

1.01 Write equivalent forms of algebraic expressions to solve problems.
   a) Apply the laws of exponents.
   b) Operate with polynomials.
   c) Factor polynomials.

A. Play Krypto (B-18) with students. A deck of Krypto cards is described as follows: three of each number 1-10; two of each number 11-17; one of each number 18-25. Randomly select five cards. All five of these numbers will be combined, using operations (including exponents and roots), to make an expression equal to your target number, a sixth card chosen randomly. With multiple sets of cards, groups of students can play. Example: Suppose 23 is the target number and 7,8,4,3, and 16 are the numbers to work with.

   \[ 23 = (16 + 4 + 3) \cdot (8 - 7) \]
   or
   \[ 23 = 8 \cdot 3 - (\sqrt{16} + 4)^7 \]

B. Roll seven dice (five WHITE, one RED, and one GREEN). The outcomes of the WHITE dice must combine correctly, using any operations, to equal 10 \cdot GREEN + RED.
   Example: GREEN = 4, RED = 6, WHITES = 6, 4, 4, 3, 1
   One solution is 46 = 6 \cdot (4 + 3) + 4 \cdot 1

C. Four in a Row
   Students will need game boards (B-22), markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicted factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another.
D. In early April, use the **Income Tax Rate Schedules (B-69)** to have students calculate taxes.

E. Develop or have the students develop flowcharts to simplify expressions.

Example: Choose \( x \).

```
\[ x \]
\[ \text{add 6} \]
\[ \text{multiply by } \frac{5}{2} \]
\[ \text{add 2} \]
\[ \text{stop} \]
```

Write the expression and solution.

F. Investigate advertisements that involve percent and use the information to create problems. For instance, a car dealer offers two discounting methods. (1) Take $2000 off of the original price, then 10% more, or (2) take 10% off of the original price, then $2000 more. Which method would give the lower price to the buyer? Explain algebraically which method is best.
G. Collect five or six articles from the newspaper that use percentages. Attach each article inside a file folder and have students write problems related to the articles. Place those in the folder. In cooperative groups, have the students work the file folder problems.

H. Have students in cooperative groups make a list of as many ways as possible to use percentages. Have groups write a problem for each use. Let students report their work to the whole class. Compile the problems and use as homework, starter problems, quiz items, or test items.

I. Determine several ways to raise a number to a power using the calculator.

J. Race to the Top (B-19, 20, 21)
Enter five numbers in the triangles along the base of the triangle. To fill in space in the row above, carry out a teacher-specified operation on the numbers in the two spaces immediately below. See the example below where addition of integers is the operation.

K. Lining Up Dominoes (A-1)
Students will make a train of dominoes by successfully simplifying an expression. Blank domino sheets (B-80) can be made available so that students can create versions of the game that practice various operational and algebraic skills throughout the year, such as evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

L. Relays (B-23, 24)
Students will work in teams. Individually team members will complete a problem share the result with the rest of the team so that a team task can be finished. The format can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

M. Use area models to explain binomial multiplication. B-57, B-58, B-59, B-60, B-61, B-62.

N. Have students to compare which is largest? 100^4 1000^3 10000^2
O. **Patterns with Exponents (B-25, 26)**
A problem set that explores patterns generated when numbers are exponentially increased.

P. **I Have … Who Has …(A-3)**
Students will listen, perform operations, and respond when appropriate in a round-robin format. Students will need to be able to complete operations with integers using paper and pencil. Students will use the format to create their own versions of the activity. The format can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials that contain roots, solving a variety of equations and inequalities, and operating with polynomials.

Q. **Play match game with cards matching the expansion or answer with the number raised to a power.**

\[
\begin{align*}
5^2 &= 5 \cdot 5 &= 25 \\
n^3 &= n \cdot n \cdot n
\end{align*}
\]

R. **How Do They Fit? (A-11)**
Students will assemble a $3 \times 3$ array of puzzle pieces so that adjacent sides match mathematically. Students will be expected to create their own puzzles and have the teacher share those with the class throughout the remainder of the school year. Well constructed and edited student puzzles will provide the teacher a pool of materials to use thereafter. The format can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

S. **Scientifico (A-5)**
Students practice translating scientific notation numbers into standard notation. Students take turns rolling three dice and constructing a number in scientific notation. Ex. 3, 6, 4 can be written $3.6 \times 10^4$. After recording this number on the recording chart, the student places a marker in the proper place on the game board. The student who can make three numbers in a row, column, or diagonal is the winner.
T. Use base ten blocks to show the students that the square root is one “side” of a square.

Perfect squares makes a square. You can approximate the square root by finding a “close” square.

U. Getting to the Root of the Number (A-35)
Working in pairs, students will use base-10 blocks to build incomplete squares that represent an approximate value for a specified irrational number.

V. Bull's Eye (B-29)
The game is played with two or more people in which each player tries to reach a specified goal number with the least number of rolls of a pair of dice. Decide who goes first. If there are more than two people playing, proceed in a clockwise manner. The teacher assigns the goal number. On each student’s turn the student will: roll the dice and compete the sum (difference, product, quotient, powers, roots) of the two numbers, either add or subtract that result from his/her cumulative total, and record the proceeds on the score sheet. The winner is the person who reaches the goal number with the fewest rolls of the dice. If no one reaches the goal number after 16 rounds, the winner is the student who is closest to the goal number. Calculators may be reserved to use only when powers and roots are the operations of choice.
W. Use algebra tiles to demonstrate addition and subtraction of like terms in polynomials.

\[(2x^2 + 3x + 2) + (x^2 - 2x + 1)\]

\[
\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
+ \\
\square \\
\square \\
\end{array}
\]

\[3x^2 + x + 3 \text{ is the result.}\]

\[(x^2 - 2x + 1) - (x^2 - x + 3)\]

\[
\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\square \\
\end{array}
\]

Take away \(x^2\), \(-x\), and add zero (+2 and -2).

Now take away 3.

\[\square \]

\[(-x - 2) \text{ is the result.}\]

X. Have students write two monomials whose product is \(-20x^4y^3\). Record possible answers on the board/overhead. Have the students make generalizations about possible answers.
Y. Look at the following patterns and find the nth terms in each pattern.

<table>
<thead>
<tr>
<th>I.</th>
<th>3² - 1² = 8</th>
<th>12</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4² - 2² = 12</td>
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<td>5² - 3² = 16</td>
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<thead>
<tr>
<th>II.</th>
<th>4² - 1² = 15</th>
<th>27</th>
<th>30</th>
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<tbody>
<tr>
<td></td>
<td>5² - 2² = 21</td>
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<td>6² - 3² = 27</td>
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<thead>
<tr>
<th>III.</th>
<th>5² - 1² = 24</th>
<th>32</th>
<th>35</th>
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<tbody>
<tr>
<td></td>
<td>6² - 2² = 32</td>
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<tr>
<td></td>
<td>7² - 3² = 40</td>
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</table>

nth ?  ???  ???  ???  ???  ???  ???

I. \((n + 2)^2 - n^2 = 4n + 4\)
II. \((n + 3)^2 - n^2 = 6n + 9\)
III. \((n + 4)^2 - n^2 = 8n + 16\)

Z. After trying **Basketball: With the game on the line ... (A-25)**, try **Basketball Extension 3 (B-17)**. This activity deals with generating polynomials that model free-throw shooting. The **Basketball** activities generally connect probability and statistics with algebra.

AA. Model multiplying monomials with chips on the overhead. After modeling several examples, have students discuss rules they could use to multiply monomials. Ask students to discuss how this rule differs from adding two monomials.

\[2x^2y \cdot 3x^3y^2 = \]
\[2 \times x \times y \times 3 \times x \times x \times y \times y\]

\[2 \times 3 \times x^5 \cdot y^3 = \]
\[2 \times 3 \times x \times x \times x \times x \times y \times y \times y\]

\[= 6x^5y^3\]

BB. Divide students into pairs. Have pairs design a puzzle to match problems with monomial • monomial and solution. Students can cut up puzzle, place in an envelope to exchange with another to solve.

CC. Have students write two monomials whose product is \(-20x^4y^3\). Record possible answers on the board/overhead. Have the students make generalizations about possible answers.
DD. **Multiplication With Algebra Tiles (A-29)**

Working in pairs, students will use pairs of binomials as the dimensions of a rectangle. The students will use the algebra tiles to build rectangles of given dimensions and find the area of the rectangle, the product of the binomials.

EE. Use the matrix method to multiply binomials (B-50, 52). Make appropriate connections with multiplying using algebra tiles.

\[
\begin{array}{c|c|c}
& x & 3 \\
\hline
x & & \\
\hline
7 & & \\
\end{array}
\]

For \((x + 3)(x + 7)\)
Take the term from the first row and multiply it with each term at the top of the matrix and place the products in the appropriate spaces in the first row.

\[
\begin{array}{c|cc}
\times & x & 3 \\
\hline
x & x^2 & 3x \\
7 & 7x & 21 \\
\end{array}
\]

Do the same with the term from the second row and place the products in the appropriate spaces in the second row.

\[
\begin{array}{c|cc}
\times & x & 3 \\
\hline
x & x^2 & 3x \\
7 & 7x & 21 \\
\end{array}
\]

In the matrix (table) of the products, add along the diagonal from right to left.

\[
(x + 3)(x + 7) = x^2 + (3x + 7x) + 21 = x^2 + 10x + 21
\]
FF. Polynomial Four in a Row (B-55)

Students will need game boards, markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another. A blank game board for Four in a Row is provided (B-56) so that teachers can give students the opportunity to create their own versions and address specifically other objectives in Algebra.

GG. Before using algebra tiles, demonstrate binomial multiplication using an area model. See B-57, 58, 59, 60, 61, 62.

HH. Operating With Binomials (A-33)

Students will fill in the entries for \(Y_1\) and \(Y_2\) with binomials and , using the calculators, determine and record the graphs of the products of binomials. Students are expected to identify the solutions (x-intercepts) of linear and quadratic equations for each graph in the matrix. Students can use a similar process to explore the sums, differences, products, and quotients of varying degrees of polynomials.

II. Use the matrix (table) method to multiply polynomials. Fill in the space at the top of each column with the terms from one of the polynomials. Fill in the space at the left side of the matrix for each row with the terms from the other polynomial.

For \((2x^2 + 3x - 4)(3x + 5)\)

\[
\begin{array}{c|c|c|c}
\bullet & 2x^2 & 3x & -4 \\
3x & & & \\
5 & & & \\
\end{array}
\]

Take the term from the first row and multiply it with each term at the top of the matrix and place the products in the appropriate spaces in the first row.

G-10
Do the same with the term from the second row and place the products in the appropriate spaces in the second row.

\[
\begin{array}{c|ccc}
\bullet & 2x^2 & 3x & -4 \\
3x & 6x^3 & 9x^2 & -12x \\
5 & 10x^2 & 15x & -20 \\
\end{array}
\]

In the matrix (table of products), add along the diagonals from right to left.

\[
\begin{array}{c|ccc}
\bullet & 2x^2 & 3x & -4 \\
3x & 6x^3 & 9x^2 & -12x \\
5 & 10x^2 & 15x & -20 \\
\end{array}
\]

\[
(2x^2 + 3x - 4)(3x + 5) = \\
= 6x^3 + 9x^2 + 10x^2 - 12x + 15x - 20 \\
= 6x^3 + 19x^2 + 3x - 20
\]
JJ. **The Month of Algebra (A-27)**
In pairs, students will select several sets of dates in the month of Algebra and complete computations according to teacher directions. By replacing the numbers in a set with appropriate variable expressions and repeating the computations, students will be able to justify algebraically the pattern.

KK. **Algebra Uno**
Use sets of 100 index cards marked like the layout (B-63) of suggested cards (include four wild cards). The game is for three or four players and proceeds much like regular “Uno”. In order to lay down a card, the player’s card must show a term that has a common factor, other than one, card facing up on the discard pile. The player who goes out first wins. A more challenging version could include polynomials on the cards.

LL. Use the matrix (table) method to factor the difference of two squares (B-51, 52).

MM. Have students reverse the multiplication process to find two binomials that multiply to give these answers. (a) \(x^2 - 25\), (b) \(a^2 - 16\), (c) \(4y^2 - 16\). What pattern do they notice? Have the students write their rule for finding two binomials that have a product called the difference of two squares. Discuss how factoring is the process of reversing multiplication.

NN. Demonstrate \(a^2 - b^2 = (a + b)(a - b)\). See B-64
Give directions orally. Cut a square and label the sides \(a\). What is the area of this square? Draw a smaller square in a corner of the first square and label its sides \(b\). What is the area of this second square?
Cut off the smaller square from the corner of the larger square. What is the area of this figure?

Cut the figure as shown. Rearrange these two new pieces so that they form a rectangle.

What is the area of this new rectangle? Have the students discuss the length and width of the rectangle. Ask the students to write an analysis of how this demonstrates the difference of two squares.
For a quadratic expression of the form \( ax^2 + bx + c \) (where \( b \) can equal zero), place the \( ax^2 \) term in the upper location (first row, first column) and the \( c \) term in the lower right (second row, second column).

What are two possible factors of \( ax^2 \)? Place the factors in the locations along the boundary for the first row and the first column.
What are two possible factors of \( c \)? Place the factors in the locations along the boundary for the second row and the second column.

Multiply the binomials that are now in the boundary. Do the two diagonal products add up to box? If not, go back and adjust the factors for \( c \) and/or \( ax^2 \)?

**PP. Factoring Trinomials with Algebra Tiles (A-31)**

Working in pairs, students will select algebra tiles corresponding to the terms of a given quadratic trinomial. The students will create a rectangular arrangement with the tiles and identify the dimensions of the rectangle. Each dimension will be one of the algebraic factors of the original trinomial.

**QQ.** Have students review types of factoring by writing about the relationship of factoring to multiplication. Use an example from each type of factoring that we have studied. Explain the relationship between multiplying the factors to obtain a polynomial and factoring the polynomial into the factors.
RR. Fill in each square with a digit 0 – 9. You may use each digit only once (B-65, 66). Similar puzzles are available commercially.

\[
\begin{align*}
x^2 - x - &\quad \square = (x + 2)(x - \square) \\
x^2 - 1 &\quad \square x + 1 \quad \square = (x - 2)(x - \square) \\
x^2 + &\quad \square x + 1 \quad \square = (x + \square)(x + 2) \\
x^2 - &\quad \square x - 24 = (x - 6)(x + \square) 
\end{align*}
\]

TT. The difference of two squares can be used with mental math. To multiply 24 and 26, the product can be found using \((25 - 1)(25 + 1) = 625 - 1 = 624\). Have the students try \(13 \cdot 17\). Using your overhead calculator, enter \(y = x^2\). Set your table generator to begin at ten and increment by five (or students can do this with individual calculators). Show the table of squares to the class. Have each student write a multiplication problem that would use the difference of two squares and one of the squares in the table. Take up the cards and form two teams. Read a problem and call on a person to give the product using only mental arithmetic. Each correct problem is worth one point.

UU. Have the students investigate the relationship of factoring the difference of two squares and the graph of a quadratic function.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Factors</th>
<th>Vertex</th>
<th>x-intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = x^2 - 1)</td>
<td>((x+1)(x-1))</td>
<td>(0, -1)</td>
<td>(-1, 0), (1, 0)</td>
</tr>
<tr>
<td>(y = x^2 - 4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y = x^2 - 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y = x^2 - 16)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
VV. Make the Message (B-67)
Factor quadratic and rational expressions to create a message. Using the same format, create similar messages, copy, and assign for homework, review, or extra credit.

WW. Use the game formats outlined on E-19, E-20, E-21 to practice skills and concepts with real numbers and to practice skills and concepts with polynomials. Encourage students to create their own versions of popular (legal, of course) games.

XX. Use the puzzle formats of How Do They Fit? (A-11), Lining Up Dominoes (A-1), “I Have … Who Has …” (A-3), and Relays (B-23, 24) to create puzzles for student use. Whenever possible, let students create the puzzles. These formats can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

YY. Number Crunching With Ease! (B-27)

ZZ. Calculator Tips (W-1, 3, 5, 9, 11)

AAA. Connections(W-1, 5, 7, 9, 19, 33, 35, 41, 53, 57, 63)

BBB. Warm Ups (W-2, 4, 6, 8, 10, 12, 14, 16, 22, 26, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72)

CCC. Challenges (W-2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32, 38, 42, 44, 48, 50, 52, 58, 64)

DDD. Extra Essentials (E-22, 23, 24, 25, 26, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 44)
A. What is the length of a rectangle that has an area of 12.1 cm$^2$ and a width of 5 cm?

B. The drip rate for intravenous fluids can be determined by using the formula $D = \frac{V}{4T}$

where D is the drip rate in drops per minute, V is the prescribed volume of fluid in cubic centimeters, and T is the prescribed amount of time for the fluid to be given to the patient in hours. If a doctor orders 1000 cc of fluid be given to a burn victim in 4 hours, what should the hospital staff adjust the IV drip rate to?

C. The following sequences are defined using recursion formulas. Write the first five terms of each sequence.

a. $a_1 = 5$ and $a_n = a_{n-1} + 7$ for $n \geq 3$

b. $a_1 = 4$ and $a_n = 3a_{n-1}$ for $n \geq 2$

c. $a_1 = 6$ and $a_n = 2a_{n-1} + 12$ for $n \geq 1$

D. A deposit of $10,000 is made in an account that earns 8% interest compounded quarterly. The balance in the account after n quarters is given by the sequence

$$a_n = 10,000 \left(1 + \frac{0.08}{4}\right)^n, n = 1, 2, 3,...$$

Find the balance in the account after ten years. Round to the nearest cent.

E. You buy a new car for $28,000. At the end of n years, the value of the car is given by the sequence $a_n = 28,000 \left(\frac{3}{4}\right)^n$. Find $a_3$ and write a sentence explaining what this value represents. Describe the nth term of the sequence in terms of the value of your car at the end of each year.

F. You are considering two job offers. The first company will start you at $22,000 annually and guarantee a raise of $2800 per year. The second company will start you at a higher salary, $28,000 annually, but will only guarantee a raise of $1100 per year. Find the total salary that each company will pay you over a fifteen-year period. Which company will pay the greater salary?

G. A theater has 20 seats in the first row, 22 seats in the second row, increasing by 2 seats per row for a total of 30 rows. How many seats are there in the theater?

H. Teachers’ earnings n years after 1989 can be described by $a_n = 1472n + 29,060$.

According to this model, what will teachers earn in 2089? Describe two possible situations that would make this predicted salary inaccurate.
1.03 Model and solve problems using direct variation.

A. Write an equation that expresses each relationship. Use \( k \) as the constant of variation.
   a. \( c \) varies directly as \( p \)
   b. \( r \) is directly proportional to \( t^2 \)
   c. \( w \) is directly proportional to \( y^3 \)
   d. Charles’s Law states that the volume \( V \) of an enclosed ideal gas at a constant pressure varies directly as the absolute temperature, \( T \).

B. Determine the constant of variation for each stated condition.
   a. \( c \) varies directly as \( p \), and \( c = 45 \) when \( p = 3 \)
   b. \( r \) varies directly as \( t^2 \), and \( r = 80 \) when \( t = 4 \)
   c. \( w \) varies directly as \( y^3 \), and \( w = 16 \) when \( y = 2 \)

C. \( y \) varies directly as \( x \). \( y = 35 \) when \( x = 5 \). Find \( y \) when \( x = 24 \).

D. Does the following table represent a direct-variation relationship? Why or why not?

<table>
<thead>
<tr>
<th>( x )</th>
<th>40</th>
<th>20</th>
<th>16</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

E. A person’s wages, \( w \), vary directly as the number of hours worked, \( h \).
   a. Write an equation that expresses this relationship.
   b. For a 40-hour work week, Debra earned $1600. Substitute this information into the equation from part (a) and find \( k \), the constant of variation. Then write the equation that describes Debra’s wages in terms of the number of hours she works.
   c. Use the equation from part (b) to find Debra’s wages for 25 hours of work.

F. The cost of an airplane ticket varies directly as the number of miles in the trip. If a 3000-mile trip costs $525, what is the cost of a 650-mile trip?

G. The distance an object falls varies directly as the square of the time it has been falling. If an object falls 150 feet in 3 seconds, how far will it fall in 5 seconds?

H. The power \( P \) (in watts) produced by a windmill is proportional to the cube of the wind speed \( v \) (in mph). If a wind of 10 mph generates 25 watts of power, how much power is generated by winds of 25, 35, and 65 mph? Make a table and explain the pattern.

I. In your own words, explain how to solve a direct variation problem?
Geometry and Measurement
The learner will describe geometric figures in the coordinate plane algebraically.

2.01 Find the lengths and midpoints of segments to solve problems.

A. Highways and the mile markers on the sides of highways can represent lines and segments. Therefore, the distance between mile markers is representative of the length of a segment. Using this information, answer the following question: If Mary sees mile marker 135 as she enters the highway and mile marker 98 when she exits, how far did Mary travel?

B. Use the figure at the right to answer the following.
   - Find the length of $CE$.
   - Find the midpoint of $BE$.
   - What is the coordinate of the midpoint of $AC$?

C. U is the midpoint of $ST$, V is the midpoint of $SU$, W is the midpoint of $SV$, X is the midpoint of $VW$, Y is the midpoint of $WX$, and Z is the midpoint of $TV$. If $UV = 32$, find $ZY$.

D. S is the midpoint of both $TQ$ and $PR$. If $SP = 14$ and $PR = 7x - 28$, find $x$, $RP$, and $RS$.

E. If L is the midpoint of $GM$ and $GL = 3y + 18$ and $ML = 9y - 6$, what are the measures of $LG$ and $GM$?

F. If a triangle has vertices $(5, -1)$, $(-2, -2)$, and $(3, 8)$, find the perimeter of the triangle and the midpoint of each side of the triangle.
2.02 Use the parallelism or perpendicularity of lines and segments to solve problems.

A. The following picture is a screen from the TI-83/84 showing Quadrants I and II, a fixed line with two points marked, and line \( Y_1 = AX + B \) with \( A = 0 \) and \( B = 5 \).

Look at the screen and answer the following questions:

- How would you change the value of \( A \) to make the two lines the same?
- How would you change the value of \( A \) to make the two lines parallel?
- How would you change the value of \( A \) to make the two lines perpendicular?

B. If \( Y_1 = 5X + 10 \), write equations for \( Y_2 \) and \( Y_3 \) so that their graphs are parallel and perpendicular, respectively, to the graph of \( Y_1 \). Graph the lines.

C. Write the equations of lines parallel and perpendicular to \( 3x + 7y = 16 \).

D. Exploring Perpendiculars Draw several segments on graph paper and give each student a copy (B-43). Have the students find the slope of each segment and record. Have the students fold the graph paper so that the fold is perpendicular to the original line. (This can be done by matching the endpoints and folding.) Have the students find the slopes of the folds and compare the results with the slopes of the corresponding segments.
Data Analysis and Probability
The learner will collect, organize, and interpret data with matrices and linear models to solve problems.

3.01 Use matrices to display and interpret data.

A. National League Matrix (1997 season through July 13, from USA Today)
The rows (top to bottom) and columns (left to right) are arranged alphabetically by teams: Atlanta, Chicago, Cincinnati, Colorado, Florida, Houston, Los Angeles, Montreal, New York, Philadelphia, Pittsburgh, St. Louis, San Diego, San Francisco. A team’s number of victories against each opponent can be found by reading across. The number of losses can be found by reading down. What is Atlanta’s overall record against the other National League teams?

<table>
<thead>
<tr>
<th></th>
<th>At</th>
<th>Ch</th>
<th>Ci</th>
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<th>NY</th>
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<tr>
<td>At</td>
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<td>4</td>
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<td>7</td>
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<td>4</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

B. Use the NHL standings from the newspaper to create a matrix that reports points (based on wins and ties), goals scored, and goals allowed. Based on the matrix, how do points correlate with goals scored? Goals allowed? If a team had 25 wins, how many goals would it be expected to score? Allow?

C. An almanac is a great source of data to use in matrix formats. For instance, use the Average Daily Temperatures in Tourist Cities from the Information Please Almanac to create matrices for North America, Europe, and the Southern Hemisphere. Compare temperature variations among the cities. Compare the data between Europe and the Southern Hemisphere; differentiate between the two sets. Compare data between cities close to the equator and those not. Find similar information about major US cities and report in a matrix. Do the same for NC cities.
D. Use the matrix which accompanies the NC maps published by the NC Department of Transportation to determine distances between NC cities. Use the matrix to estimate driving distance for a trip to the NC mountains during Leaf Season. One such trip may follow this route for a round trip from Raleigh. Raleigh – Greensboro – Charlotte – (via US 74 and I-26) - Asheville – Boone – Winston-Salem – Raleigh.

E. Create and update a matrix for ACC basketball, through the ACC tournament, similar to the one for major league baseball.

F. Use the tax tables in the current IRS 1040 Forms and Instructions or NC Individual Income Tax Forms and Instructions (B-69) tax materials to determine taxes owed for varying amounts of taxable income. Describe how the tax tables are organized (income span per row, relationships among filing categories, how does the tax owed change as the income increases, identify similarities and differences between the state and federal tables, etc.). Compare the tables with the actual tax rate schedules for both forms.

G. Have students bring in prices of a large pizza with one topping, large drink, and extra topping from three or four favorite pizza restaurants. Set up a 3 • 4 matrix. Here is one example:

<table>
<thead>
<tr>
<th></th>
<th>Italian Delight</th>
<th>Dot’s Pizza House</th>
<th>Round Romans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Pizza</td>
<td>$8.99</td>
<td>$11.40</td>
<td>$12.20</td>
</tr>
<tr>
<td>Large Drink</td>
<td>.99</td>
<td>.85</td>
<td>.90</td>
</tr>
<tr>
<td>Extra Topping</td>
<td>1.30</td>
<td>1.25</td>
<td>1.50</td>
</tr>
</tbody>
</table>

H. Have students investigate information from two matrices. For example: Pete investigates interest rates at First Bank and Credit Savings for certificates of deposit (CD) and money market accounts. This information is listed in the 2 • 3 matrix.

<table>
<thead>
<tr>
<th></th>
<th>12 CD</th>
<th>48 CD</th>
<th>MM</th>
<th>12 CD</th>
<th>48 CD</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Bank</td>
<td>5.8%</td>
<td>6.8%</td>
<td>4.8%</td>
<td>Option I</td>
<td>$15,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>CreditSav</td>
<td>6.5%</td>
<td>6.5%</td>
<td>5.4%</td>
<td>Option II</td>
<td>$10,000</td>
<td>$25,000</td>
</tr>
</tbody>
</table>

What would be the interest for one year on a 48 month CD for option I at First Bank? What would be the total interest for option I at Credit Savings? Now compare this to option I at First Bank? Compare the total interest for option II at both banks. Which investment option would be the best for Pete?

I. Using the matrix for In-Line Skating (B-30), add a row that estimates calories consumed for a 130 pound skater. As speed increases how does the number of calories consumed at each weight change? Do the calories increase at the same rate at every weight? Using the matrix for Personal Fitness (B-31), add a row that estimates calories consumed by a person running 13 minute miles. For a person weighing 130 pounds, how many calories does he consume in each activity?
J. Determine the top 15 countries that earned gold, silver, and bronze medals at the 2002 Olympics. Organize a matrix to display the information. Order the data so that there are columns for gold, silver, and bronze medals won. Order the rows according to the country that won the most gold medals at the top of the matrix and the country that won the least gold medals at the bottom. If two or more countries had the same number of gold medals, use the most silver medals, and then bronze if needed. Add a fourth column with the medal totals. The matrix can be expanded by adding a column that shows the most recent census or population estimate of each country. In another column record how many medals were won per thousand people in each country. Analyze and discuss any trends or relationships that you notice in the matrix.

K. Have students set up a matrix of sports data for their favorite team (see B-15 for an example) on a spreadsheet. Have them enter at least three formulas to calculate averages or totals for the players or the team. How could the matrix (B-15) be organized to better investigate correlations between minutes played and points scored or free throw attempts and rebounds? If a player had 1700 minutes of playing time, how many points should he have been expected to score? If a player collected 250 total rebounds, how many free throws would I have expected him to attempt?

L. Have students set up a matrix of data on the planets of our solar system on a spreadsheet. Enter columns for equatorial diameter, mean solar distance, and orbital period. Use the spreadsheet to create new columns in the matrix which compare planetary diameters with Earth’s and mean solar distance in astronomical units. Look for patterns in the data and potential correlations (mean solar distance and orbital period) by reordering rows according to mean solar distance, equatorial diameter, etc.

M. Connections (W-3, 11, 15, 17, 35, 43, 49, 51, 61, 63, 67, 71)
3.02 Operate (addition, subtraction, scalar multiplication) with matrices to solve problems.

A. Given the following matrices:

\[
[A] = \begin{bmatrix} 32 & 34 & 46 \\ 24 & 25 & 34 \\ 30 & 38 & 49 \end{bmatrix}, \quad [B] = \begin{bmatrix} 10 & 20 & 30 \\ 10 & 20 & 20 \\ 20 & 20 & 25 \end{bmatrix}, \quad [C] = \begin{bmatrix} 60 & 60 & 60 \\ 50 & 50 & 50 \\ 44 & 44 & 44 \end{bmatrix}
\]

Find:
- [A] + [B]
- 2[A]
- [A] + \(\frac{1}{2} [C]\)
- [C] - [B]

B. The Booster Club at a local high school sells food, candy, drinks, megaphones, hats, and rally towels during home sporting events. The following matrices show the total sales, cost of supplies, and operating expenses for the football, basketball, and baseball seasons during 2004 and 2005. Find the total sales, costs of supplies, and operating expenses for the Booster Club.

<table>
<thead>
<tr>
<th></th>
<th>Football</th>
<th>Sales</th>
<th>Cost of Supplies</th>
<th>Operating Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>$19,208</td>
<td>$8,211</td>
<td>$2,609</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>$21,553</td>
<td>$10,420</td>
<td>$2,511</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Basketball</th>
<th>Sales</th>
<th>Cost of Supplies</th>
<th>Operating Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>$13,174</td>
<td>$6,426</td>
<td>$2,540</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>$12,845</td>
<td>$5,315</td>
<td>$2,187</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Baseball</th>
<th>Sales</th>
<th>Cost of Supplies</th>
<th>Operating Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>$9,428</td>
<td>$6,201</td>
<td>$1,807</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>$11,206</td>
<td>$7,005</td>
<td>$1,775</td>
<td></td>
</tr>
</tbody>
</table>
3.03 Create linear models for sets of data to solve problems.
   a) Interpret constants and coefficients in the context of the data.
   b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

A. Have students graph data from an almanac or other source. Teacher to Teacher (W-51) identifies several sources for useful data, see B-30 and B-31 for some examples. Write an equation to model the relationship.

B. Take advantage of data that appear in a newspaper or magazine. Here is an example. In 1986, the 20th Super Bowl was played in New Orleans. The price of a 30-second television commercial for the game was $550,000. In 1993, the 27th Super Bowl was played in Pasadena and the price of a 30-second commercial had risen to $850,000. Write a linear equation that models the change in the price of television commercials for this sporting event.

C. Scoring and Winning (A-7) Have students gather data from the NFL (or similar data from the NBA, NHL, MLB, or local minor leagues) to create scatter plots and find lines of best fit. Students will discuss the characteristics of those lines and make predictions.

D. Find the flight times and direct air distances from the larger North Carolina airport (Charlotte, RDU, Piedmont Triad, etc.) near you. Use airline tables and be aware of time zone considerations as flight times and air distances are compiled. Plot the data, find the line (or other curve) of best fit. Discuss the meaning of slope and x- and y-intercepts in the context of the data.

E. The Good Estimator (A-9) Individual students or groups will estimate ten distances in the classroom or other parts of the school. Then the students will measure the same distances and compare their estimates with the “real” measures.

F. The Wave (A-13) In a whole class setting, increasingly larger groups of students will perform the “wave”. The students will collect and interpret data, determine a linear function of the time to complete the “wave” dependent upon the number of students participating, and use the linear function to make predictions.

G. Patterns in Perimeter (B-32, 33, 34, 35) Students will use perimeter to generate data, find the algebraic expression for the linear pattern, and graph the data that are generated. Although it is expected that students will work individually or in pairs on the pattern sheets, overhead versions of two of the problems are provided for whole class discussion. Using pattern blocks to build the figures is particularly helpful for many students’ understanding.
H. After trying Basketball: With the game on the line ... (A-25), try Basketball Extension 1 (B-13). The Basketball activities generally connect probability and statistics with algebra.

I. Used Cars Have students ask 10-12 teachers the ages and mileages for their cars. Have the class draw a scatter plot and a line of best-fit. Ask students to estimate a slope and interpret the slope in the context of the situation.

J. Give the equation $y = 3.5x + 25$, have students create a situation modeled by the equation. For instance, let the equation describe the relationship between the number of martial arts classes ($x$) and their cost ($y$) at the YMCA. What does the $y$-intercept represent? What does the slope represent?

K. Connecting Units of Measure (A-21) Students will measure several objects in the classroom using both centimeters and inches. Students will plot corresponding pairs of measurements on a graph and interpret the information to determine the relationship between centimeters and inches.

L. Making Sense of Slope The handouts B-44, 45, 46 contain five problems that are related to graphing and slopes of lines. Each problem contains a graph or chart that the students are to analyze and answer questions about. Work problem #1 as a class to give students a good idea of what is expected. It would be appropriate for students to work in groups of three or four to complete the assignment.

M. Patterns in Area and Volume (B-36, 37, 38) Students will use surface area and volume to generate data, find the algebraic expression for the pattern, and graph the data that are generated. Using blocks to build the figures is particularly helpful for many students’ understanding. There are linear, exponential, and quadratic patterns among the sequences. Only after students have described the patterns in arithmetic terms should students use the curving fitting utilities on their calculators. The method of finite differences can be used to identify quadratic patterns. (With Algebra II and Technical Math 2 students, follow up finite differences by setting up the matrix equation and solve it to derive the quadratic expression.)

N. After students have created an equation based on data, have them use it to make predictions. Here is a continuation of the example from 3.03B. What was the price of a 30-second television commercial in 2000? What was it actually? Explain any differences. Do the same for the first Super Bowl in 1967.

O. According to postal rate information, B-47, what will be the cost of a stamp in the year 2010? In what year do you predict stamps will cost 50¢?

P. It’s All Downhill From Here (A-19) Create a linear model to represent rolling a ball down a ramp. Use the model to predict how far the ball will roll down a ramp of given height.
Q. Students can use the internet to find data on housing costs to graph. Have students look up data at www.realtor.com or go through their local chamber of commerce web site. They can make a table for ten houses listing the square footage of a house and the cost of a house. Using a calculator, students can draw a scatter plot of the data and find the line of best fit. Have students define and compare the slope and y-intercept for different cities or different locations in one large city. (Most often the slope would be the average cost per square foot and the y-intercept is the average price of the lot.)

R. Give students a table of data. Ask them to determine the slope and interpret. Example:

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Wages Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>28.75</td>
</tr>
<tr>
<td>10</td>
<td>57.50</td>
</tr>
<tr>
<td>15</td>
<td>86.25</td>
</tr>
<tr>
<td>20</td>
<td>115.00</td>
</tr>
</tbody>
</table>

S. How Do You Measure Up? (A-15) Students will measure their forearm and height. The collected data will be organized, interpreted, and used to make predictions.

T. What Shape Are You? (A-17) In groups, students will measure their height and arm span. They will graph their height vs arm span to investigate data and make predictions.

U. Toothpick Triangles (A-23) This activity will have students building sets of triangles with toothpicks. Students will use the toothpicks to visualize patterns and use the data generated by the pattern to generate a linear relation and make predictions.

V. Creating “Real World” Problems (B-9) Students will create linear equations and make predictions.

W. Calculator Tips (W-35, 39, 43)

X. Connections (W-49)

Y. Extra Essentials (E-23, 24, 35, 43)
Algebra
The learner will use relations and functions to solve problems.

4.01 Use linear functions or inequalities to model and solve problems; justify results.
   a) Solve using tables, graphs, and algebraic properties.
   b) Interpret constants and coefficients in the context of the problem.

A. Develop an experiment to send a toy car down a ramp on various heights. Clock the time and determine the height of the ramp. Collect data, make graphs, find the line of best-fit, and interpret the slope and y-intercept in the context of the problem. Make predictions of time traveled based on changing heights of the ramp.

B. The Picture Tells the Linear Story (B-39, 40, 41)
   Students investigate families of linear equations.

C. Moving on the Graph (B-42)
   What do your students know about transforming a graph?

D. Connections problems (W-23, 53)

E. Drive Time problem (B-68)

F. Used Cars problem
   Have students ask 10-12 teachers the age and mileage (odometer reading) for their cars. Have the class determine the line of best-fit, and interpret the slope and y-intercept in the context of the problem.

G. Give the equation \( y = 3.5x + 25 \). Have students create a situation modeled by the equation. For instance, let the equation describe the relationship between the number of martial arts classes (\(x\)) and their cost (\(y\)) at the recreation center. What does the y-intercept represent? What does the slope represent?

H. Use a relay format to practice solving inequalities.

I. Select an inequality (example: \( x - 5 < 2 \)). Enter each side of the inequality on the calculator (\( Y_1 = x - 5 \) and \( Y_2 = 2 \)). Graph, locate the intersection, and identify for which values of \( x \) is the left side above (greater) or below (less) the right side.
J. **Placekicker**
In football, the place kicker can score points two different ways. He can score 3 points with a field goal or 1 point with a point-after-touchdown (PAT). The coach expects his kicker to score at least 50 points during the season. How many field goals and PATs could the kicker score?

K. Select an inequality (example: \(-2 < x + 7 < 6\)). Enter each part of the inequality on the calculator \((Y_1 = -2, Y_2 = x + 7, \text{ and } Y_3 = 6)\). Graph, locate the intersections, and identify for which values of \(x\) is the center part between the other two parts. Solve by hand and compare results with the calculator results.

L. Divide the students into teams. Have the students use any method they wish (including a calculator) to solve the equation. Keep score. Reward appropriately the team with the most correct solutions. Be sure to divide the teams equitably.

M. **Re-solving Equations (B-132, 133)**
Students are given several simple linear equations that have been solved incorrectly. Students are expected to identify and explain errors and then solve the equation correctly. Use the blank setup \((B-134)\) to adjust the exercises according to topics being studied.

N. Select a linear inequality. (example: \(4x + 2y \leq 6\)) Solve for \(y\) in terms of \(x\) \((y \leq -2x + 3)\). Enter the corresponding linear equation in the calculator and graph. Identify the region above (greater) or below (less) the line as the solution set. Using the appropriate calculator function, \((W-21, W-23)\) graph and record. Choose several points in the region to check in the original inequality. Have the students use this process with several inequalities.

O. Use algebra tiles to demonstrate solving simple equations using the additive inverse.

Let students use tiles to solve similar problems generated from the teacher or textbook.

For example:

\[
\begin{array}{ccc}
\square & \square & \square \\
\square & \square & \square \\
\end{array}
= \\
\begin{array}{ccc}
\square \\
\square \\
\end{array}
\]

\(2x + 5 = x + 2\)
Use algebra tiles to demonstrate solving simple equations using the multiplicative inverse. Let students use tiles to solve similar problems generated from the teacher or textbook. For example:

Add -5 to each side of the equation and simplify.

\[2x = x + -3\]

\[=\]

\[=\]

Add -x to each side of the equation and simplify.

\[=\]

\[x = -3\]

P. Use algebra tiles to demonstrate solving simple equations using the multiplicative inverse. Let students use tiles to solve similar problems generated from the teacher or textbook. For example:

\[3x + 2 = x + 6\]
Add -2 to each side of the equation and simplify.

\[ \boxed{3x = x + 4} \]

Add \(-x\) to each side of the equation and simplify.

\[ 2x = 4 \]

Separate each side of the equation into two equal parts.

\[ x = 2 \]
4.02 Graph, factor, and evaluate quadratic functions to solve problems.

A. Have students hand-graph examples of quadratic equations that are in the textbook. Pair students and ask them to generate a list of characteristics of each equation and quadratics in general.

B. Have students graph \( y = 2x - 1 \) and \( y = x^2 - 1 \) on separate graphs. Ask them to write a description of how the graphs are alike and how they are different.

C. The Picture Tells the (Quadratic) Story (B-70, 71).
Students investigate families of quadratic equations.

D. Have students use and investigate quadratic equations that could represent physical phenomena. The quadratic equation \( y = -4.9x^2 + 30x + 1.5 \) describes in meters the height of a baseball that is traveling at 30 meters per second and has been hit from a height of 1.5 meters above the ground. (1) Make a table of values, (2) graph the equation, (3) as time increases, describe the height of the ball, and (4) how does the graph differ from a linear equation?

E. Have students graph some quadratic equations which model a situation they may be familiar with.

F. Max-Min (B-72)
Give students three points to graph on graph paper. Have them sketch in a parabola that includes the three points and estimate the maximum or minimum value for the parabola. Have the students share their results. After several examples, give students two points to work with. Again, have students share their results. How many points are needed to determine a parabola?

G. Open Boxes (A-39)
In groups, students will collect and analyze data from the construction of several boxes and estimate the maximum volume. Students will use an algebraic model to determine a maximum volume and compare it with the experimental results.

H. The Maximum Garden (A-41)
Students will use a table to list possible values for the dimensions and area of a garden space. Students will write an equation to graph the width of the garden verses the area. This will be a quadratic function with a maximum value.
I. Have students relate the solutions of quadratic equations to the graphs of related functions.

(1) Solve by factoring
   a. \( x^2 + x - 6 = 0 \)
   b. \( x^2 - 8x + 12 = 0 \)

(2) Now graph on the calculator the related functions and record the graphs. Where does each function cross the x-axis?
   a. \( y = x^2 + x - 6 \)
   b. \( y = x^2 - 8x + 12 \)

(3) How can the solutions to quadratic equations be found by graphing the related function? Why?

(4) Find the solutions to the nearest tenth.
   a. \( x^2 + 2x - 7 = 0 \)
   b. \( 2x^2 - 8x + 3 = 0 \)
   c. \( 4x^2 = 8x + 7 \)

J. Make an overhead of a picture of a dolphin jumping. Have students graph on a graphing calculator this equation, \( y = -0.35x^2 + 1.48x \) where \( x \) is the horizontal distance of a dolphin jump and \( y \) is the vertical distance of the jump in meters. Ask students to find how high the dolphin jumps and after how many meters will the dolphin enter the water.

K. Shuttle Launch (A-43): Working in pairs, students will use a pair of quadratic equations to identify the critical points along the flight of the solid rocket boosters (SRB) that are used to launch the space shuttle. Students will connect algebraic ideas (intersection, vertex, x-intercept, and evaluating expressions) with points along the flight path (engine shutdown, maximum altitude, splashdown, and altitude versus elapsed time).

L. Use quadratic equations to represent a commercial situation. The profit of a business can be described by the equation \( P = 1.8T^2 - 20T + 250 \), where \( P \) is the profit in thousands of dollars and \( t \) is the number of years since 1995 (\( t = 0 \) corresponds to 1995). Describe the profit trend over the last ten years. Use the model to predict the year in which the profit will be double that of 1995.

M. Have students solve equations in factored form with a product of zero from the textbook. Have them explain (1) why this method works, (2) how you can tell if a solution will be positive or negative, and (3) how you can tell if a solution will be non-integral.
N. Have the students reverse the processes. Ask them to write a quadratic equation that has (1) two positive solutions, (2) two negative solutions, (3) two solutions that are fractions, and (4) only one solution.

O. Have students discuss why the following is incorrect. Tanya solves a quadratic equation this way.

\[(2x - 1)(x + 2) = 3\]

\[2x - 1 = 3 \quad \text{or} \quad x + 2 = 3\]

\[x = 2 \quad \text{or} \quad x = 1\]

Does \(x = 2\) check as a solution to the equation?  
Does \(x = 1\) check as a solution?  
What is wrong with Tanya’s method?

P. Operating with Binomials (A-33). Students will fill in the entries for \(Y_1\) and \(Y_2\) with binomials and, using calculators, determine and record the graphs of the products of the binomials. Students are expected to identify the solutions (x-intercepts) of linear and quadratic equations for each graph in the matrix. Students can use a similar process to explore the sums, differences, and products of varying degrees of polynomials.

Q. Factoring Trinomials with Algebra Tiles (A-31). Working in pairs, students will select algebra tiles corresponding to the terms of a given quadratic trinomial. The students will create a rectangular arrangement with the tiles and identify the dimensions of the rectangle. Each dimension will be one of the algebraic factors of the original trinomial.

R. Polynomial Four-in-a-Row (B-55). Students will need game boards, markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another. A blank game board for Four in a Row is provided (B-56) so that teachers can give students the opportunity to create their own versions and address specifically other objectives in algebra.

S. Use the matrix method for factoring quadratic expressions. See 1.01 00.

T. Have students solve second-degree equations by factoring using problems from their textbook. Have students explain why the method does or does not always work. Have them write a second-degree equation that cannot be factored.
U. Give students a table of values for \( y = x^2 - 2x - 8 \).

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>7</td>
<td>0</td>
<td>-5</td>
<td>-8</td>
<td>-9</td>
<td>-8</td>
<td>-5</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

(1) From the table, find solutions when \( y = -5 \), or \( x^2 - 2x - 8 = -5 \).

(2) Now use factoring to find the solutions.

(3) Solve each of the following using the table, then factoring.
   a. \( x^2 - 2x - 8 = 7 \)
   b. \( x^2 - 2x - 8 = -8 \)
   c. \( x^2 - 2x - 8 = 0 \)

(4) Develop a table with ten ordered pairs using the function \( y = x^2 + 2x - 15 \). Write four equations you can solve using your table. Solve the four equations by factoring.

V. Use the cover-up method to have students solve equations such as \( (x + 2)^2 = 49 \). On the overhead, write \( (x + 2)^2 = 49 \) and place a square of paper over the term inside of the parentheses: \( ( \quad )^2 = 49 \). So \( (x + 2) = 7 \) or \( (x + 2) = -7 \) which means \( x = 5 \) or \( x = -9 \).

W. Use an application. Each side of a square patio was increased in length by 5 feet to give an area of 150 square feet. What was the original length of the patio? [If \( x \) is the original length, solve for \( x \) when \( (x + 5)^2 = 150 \).]

X. When given equations like \( x^2 - 3x + 4 = 7 \), have students enter each side of the equation in the calculator as \( Y_1 = x^2 - 3x + 4 \) and \( Y_2 = 7 \). Graph, trace, and zoom to locate the intersection.

Y. Using the North Carolina population information (B-47) predict the population in 2010. In what year should the population of North Carolina reach ten million? Try the quadratic curve of best fit with the calculator.

Z. Quadratic Functions (A-37) Students will find solutions to quadratic equations using two different methods, factoring the equation and graphing the related function to find the \( x \)-intercepts.

AA. Connections (W-29)
4.03 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.

A. Olympic Swimming (B-48)
With the Olympics occurring every two years (alternating summer and winter games), there are several events in which both men and women compete. For instance, there are winning results for men and women in the 400 meter free-style swimming since 1924. Working in pairs, students will select (or be assigned) an event. Using their calculators, they would determine best-fit linear equations for each of the men’s and women’s data. If appropriate, use the equations to determine men’s and women’s performances for 1940 and 1944. (Why were there no results in those years?) Predict the winning results for the next several Olympics. Ask the students to determine, according to their calculations, if the women’s performance will ever equal or exceed the men’s performance in their event. Research as to whether this is likely to happen. At the 2004 Athens Olympics the winning times in the 400 meter free-style events were 223.10 seconds for the men and 245.34 seconds for the women.

B. Have students research the transportation costs for travel between cities. Assume that the costs identified represent a linear trend. Here is an example from January 11, 2003:

<table>
<thead>
<tr>
<th>Distance (round trip)</th>
<th>Car</th>
<th>Air</th>
<th>Rail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raleigh-Charlotte</td>
<td>300 miles</td>
<td>$108</td>
<td>$224</td>
</tr>
<tr>
<td>Raleigh-New York</td>
<td>1300 miles</td>
<td>$468</td>
<td>$169</td>
</tr>
</tbody>
</table>

Determine the distance at which driving a car is less expensive than riding the train. When does it become cheaper to fly rather than drive? For what distance is the train the most expensive mode of travel? Identify some advantages and disadvantages for each mode of transportation. What other variables affect the cost of travel?

C. Select two linear equations and enter them on the calculator. (Example: \( Y1 = x - 4 \) and \( Y2 = -x + 3 \)). Graph, trace, and identify the intersection. Verify with substitution. Record the graph and coordinates of the intersection. Have students try other pairs of equations and record results. Use a friendly range for best results.

D. During the 1999 baseball season, each team in Major League Baseball played 162 games. On April 30, the Boston Red Sox had played 22 games and won 11. Meanwhile the New York Yankees had won 14 of their 21 games. By May 31, Boston had played 50 games and won 31. New York had played 48 games and won 28. Create a linear equation in slope-intercept form that describes the trend for each team. Define the slope of your equations with respect to the quantities being discussed. Explain how you created your equations and use them to predict the number of games each team should win by the end of the season. How do your results compare with the actual results for that season? Explain the difference, if any.
E. Divide students into pairs. Give each pair two systems to graph. Student A graphs the first system at the same time Student B graphs the second system. Have the pair exchange papers to find the solution to the systems from the graph and check the solutions. Each pair works together if difficulties arise.

F. Divide class into cooperative groups. Give each group three graphs of systems of equations and the tables from a spreadsheet. Have groups write the equations for each system, find the solutions from the graphs, and check the solutions using the equations.

G. Select two linear inequalities and enter the corresponding linear equations on the calculator. Using the appropriate calculator function (*Calculator Tips W-21, 23*) graph and identify the intersection. Record the graph. Have students try other pairs of inequalities and record results.

H. Survey students and create problems similar to the one that follows. Mildred makes $3 an hour babysitting and $6 an hour when she works at Wendy’s. Her parents do not want her to work more than 20 hours per week. Mildred would like to earn at least $70 a week. Write a system of inequalities that show the number of hours she could work at each job. Graph the system. Write at least four possible solutions.

I. Use information form local businesses to create problems. Here is an example. The Twin Theater charges $7 for adult tickets and $4 for children 12 or under. The theater has 470 seats. The manager wants to have a nightly income (two shows) of at least $5000. Write a system of inequalities for the number of children and adult tickets that can be sold. Write at least four possible solutions.

J. Have students write a system of inequalities whose solution set is (1) a triangle, (2) a trapezoid, (3) a kite, (4) a hexagon.

K. A business will make money if revenues exceed expenses. Discuss with students the break-even point when revenues equal expenses. Have students consider the following hamburger business. The owner pays $20,000 for the franchise and has expenses of $750 per thousand hamburgers. The price of a hamburger is $2.09. Have students write an equation to represent the cost of the business and the equation to represent the revenue. Graph the equations. Ask the students to find the number of hamburgers the business needs to sell to break even. What would be the profit if 100 thousand burgers are sold? Many franchises work this way. Have students talk with local owners and share the information with the class.

L. Use business-type situations that arise at school to create problems. Here is an example. The Silver Ratio Band wants to talk to the school principal concerning a contract to play for the Valentine dance. The group is considering three possible rates: (1) the band will charge $3 per person; (2) the school will pay the band $50 plus $2.50 per person; (3) the band will rent the fellowship hall at a local church for $125 and charge $4 per ticket. Which method would be best for the group to use? Have students write a summary of their findings.
M. Ask students to describe the advantages of each method of solving systems of equations: graphing a system of equations on a calculator and using a spreadsheet to analyze a system of equations.

N. I’m Thinking of Two Numbers
   a. I’m thinking of two numbers. Their sum is 12. The sum of the first with twice the second is 7. What are the numbers? (17, -5)
   b. I’m thinking of two numbers. Their difference is 9. The sum of twice the first and three times the second is 63. What are the numbers? (18, 9)
   c. I’m thinking of two numbers. The first is three times as large as the second. Their sum is 48. What are the numbers? (36, 12)
   d. Ask students to create their own versions of “I’m thinking of two numbers.” Collect those from the students, compile and edit, and redistribute for students to solve.

O. Give each row of students one of the following sets of equations to solve using the addition method:
   1) \( x + 2y = 3, \ 4x + 5y = 6 \)
   2) \( 2x + 3y = 4, \ 5x + 6y = 7 \)
   3) \( 3x + 4y = 5, \ 6x + 7y = 8 \)
   4) \( 4x + 5y = 6, \ 7x + 8y = 9 \)
   5) \( 5x + 6y = 7, \ 8x + 9y = 10 \)

   Call on one person on each row to give the solution. Ask students if they notice a pattern to the equations. Why would they have the same solutions? If all systems were graphs on the same axis, what would they look like?

P. Calculator Wars
   (1) Each player enters the equations for two lines into Y1 and Y2. Wise players choose equations that do not intersect on the screen.
   (2) After they exchange calculators, the students must manipulate the window so that the crossing lines appear. They must then use the 2nd CALC INTERSECTION to find the solution to the system of equations. Since the calculation will only find the intersection of the functions displayed on the screen, the students may not skip the step of manipulating windows.
   (3) Encourage the players to use the table function to get initial estimates.
   (4) The first player to show his opponent the correct solution wins the round. As always, play continues until the teacher calls time.

Q. First Wheels (B-49)
   Construct a system of equations to make a thoughtful automobile purchase.
R. Revisit absolute value equations using the substitution method and calculators. Have students graph on the calculator, \( y = |x - 2| \) and \( y = 4 \). Use the trace to find the ordered pairs for the intersection. Next have students solve the system using substitution. (i.e. \( |x - 2| = 4 \)). Ask students to explain a method for solving an absolute value equation, \( |x - 3| = 8 \) using a calculator and solving by hand.

S. **Challenges** (W-10, 40, 46, 54, 56, 60)
4.04 Graph and evaluate exponential functions to solve problems.

A. The Dinosaurs Bite the Dust (B-76)
Simulate the extinction of the dinosaurs using dice to create an exponential model. Point out to students the random nature of the dinosaurs’ demise, yet we are able to discover a mathematical model for the event. The two theories which try to explain the dinosaurs’ disappearance are provided only to the teacher. The teacher can share this information with the students or expect the students to research the topic.

Sixty-five million years ago, Earth experienced a global extinction event so severe that it defines the boundary between the Cretaceous (K) and Tertiary (T) geological periods. Causing extinctions on both the lands and in the oceans, that event is referred to as the K-T extinctions. The dinosaurs became extinct during the K-T extinctions.

Of all the theories ever devised for cause of the K-T extinctions, only two remain and they are the focus of intense scientific debate. One, the asteroid-impact winter theory created by Nobel laureate Luis Alvarez, states that a giant asteroid struck Earth 65 million years ago. It blasted dust into the stratosphere that blocked out sunlight and plunged Earth into a dark, frozen winter.

The other, the volcano-greenhouse theory originated by Dewey M. McLean, relates the K-T extinctions to a major perturbation of earth’s carbon cycle caused by the Deccan Traps Mantle Plume Volcanism in India. This was one of the greatest volcanic events in Earth history and its main eruptions began 65 million years ago. The Deccan Traps released vast quantities of the greenhouse gas, carbon dioxide (CO₂), onto Earth’s surface, trapping heat from the sun, and turning Earth’s surface into a hot, sterilizing “greenhouse.”

It has been estimated that the dinosaurs disappeared in 100-300 years. Using the function \( y = 500000 \cdot (G)^x \), where \( x \) is the number of years since the extinction event, \( y \) is the remaining dinosaurs, and \( G \ (0.01 \leq G \leq 0.99) \) is the rate of population decline, have students determine a rate of decline when there are 100 dinosaurs left after 300 years.

B. Problems of an Exponential Nature (B-77)
Many problems like these are available in textbooks. Expect students to investigate the situation in a table, using their calculators for computation. Then students can discover and discuss patterns in a graph of their data. Students can explore curve-fitting with their calculators and/or discuss how to use the \( y = a \cdot b^x \) form to model the problem.
C. **Patterns with Exponential Equations (A-45)**

Students will graph equations in which the base $b$ is a positive number greater than 1. They will investigate what happens as $b$ increases and describe the pattern. Next, students will graph equations in which the base $b$ is between 0 and 1 and describe this pattern.

D. **Rolling Dice: An Exponential Experience (B-73)**

- Begin with a large number of dice (30+). Place the dice in a cup and roll them. Remove all the dice that show 3. Roll the remaining dice and again remove the 3s. Continue the process until there are only one or two dice remaining. Keep a record of the results for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $y = N \cdot \left( \frac{5}{6} \right)^x$, where $N$ is the number of dice you begin with, $x$ is the number of rolls, and $y$ is the remaining dice?

- With a large number of dice (30+) handy, begin with two dice. Place the dice in a cup and roll them. For every die that shows a 3, add another die. Roll the dice and again add a die for each 3 that appears. Continue the process until all of the dice are used. Keep a record of the results for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $y = N \cdot \left( \frac{7}{6} \right)^x$, where $N$ is the number of dice you begin with, $x$ is the number of rolls, and $y$ is the new dice total?

E. Begin with a large number of coins (30+). Place the coins in a cup, shake, and dump on the desk top or floor. Remove all the coins that show HEADs. Shake and dump the remaining coins and again remove the HEADs. Continue the process until there are no coins remaining. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $y = N \cdot \left( \frac{1}{2} \right)^x$, where $N$ is the number of coins you begin with, $x$ is the number of rolls, and $y$ is remaining coins.

With a large number of coins (30+) handy, begin with two coins. Place the coins in a cup, shake, and dump on the desk top or floor. For every coin that shows HEADs, add another coin. Shake and dump the coins and again add a coin for each HEADs that appears. Continue the process until all of the coins are used. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function $y = N \cdot \left( \frac{3}{2} \right)^x$, where $N$ is the number of coins you begin with, $x$ is the number of turns, and $y$ is the new coin total?
F. **Use Your Imagination (B-75)**

Three problems are presented that are mental experiments involving “folding” a single sheet of paper in half many times. These are good illustrations for the power of exponential growth. (The solutions are: the Sears Tower in 23 folds; Mount Everest in 27 folds; and the Moon in 42 folds.)

G. Have students use a calculator and a table to calculate the cost of jeans at 5% inflation over a period of 5 years. Then, have them compare their yearly results to the compound interest formula. (The example shown is for a $30 pair of jeans.)

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30 + .05*30 = 31.50</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>31.50 + .05*31.50 = 33.08</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>33.08 + .05*33.08 = 34.73</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>34.73 + .05*34.73 = 36.47</td>
<td></td>
</tr>
</tbody>
</table>

Write an expression for the cost of jeans in the year 2015. Use your calculator to simplify.

H. Have students draw a graph to compare the difference in simple interest and compound interest for $2,000 invested at 6%.

<table>
<thead>
<tr>
<th>Years</th>
<th>Simple Interest</th>
<th>Compound Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000*.06*0+2000=</td>
<td>2000*.06^0</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I. **Connections (W-69)**

J. **Challenges (W-72)**

K. **Extra Essentials (E-22, 25)**

L. **Warm Ups (W-70)**