Updated 02/18/04

Integrated Mathematics 2

Integrated Mathematics 2 continues students' study of topics from algebra, geometry, and statistics in a problem-centered, connected approach. Functions, matrix operations, and algebraic representations of geometric concepts are the principle topics of study. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- ***Create linear and exponential models, for sets of data, to solve problems.***
- ***Use linear expressions to model and solve problems.***
- ***Collect, organize, analyze, and display data to solve problems.***
- ***Apply geometric properties and relationships to solve problems.***
- ***Apply the Pythagorean Theorem to solve problems.***

Updated 02/18/04

Integrated Mathematics 2

GOAL 1: The learner will perform operations with numbers, algebraic expressions, and matrices to solve problems.

1.01 Write equivalent forms of algebraic expressions to solve problems.
1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.
1.03 Model and solve problems using direct variation.
1.04 Operate with matrices to model and solve problems.

GOAL 2: The learner will describe geometric figures in the coordinate plane algebraically.

2.01 Find the lengths and midpoints of segments to solve problems.
2.02 Use the parallelism or perpendicularity of lines and segments to solve problems.
2.03 Use the trigonometric ratios to model and solve problems.
2.04 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

GOAL 3: The learner will collect, organize, and interpret data to solve problems.

3.01 Describe data to solve problems.
a) Apply and compare methods of data collection.
b) Apply statistical principles and methods in sample surveys.
c) Determine measures of central tendency and spread.
d) Recognize, define, and use the normal distribution curve.
e) Interpret graphical displays of data.
f) Compare distributions of data.
3.02 Create and use, for sets of data, calculator-generated models of linear, exponential, and quadratic functions to solve problems.
a) Interpret the constants, coefficients, and bases in the context of the data.
b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

GOAL 4: The learner will use relations and functions to solve problems.

4.01 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify steps used.
4.02 Use quadratic functions to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.
4.03 Use power models to solve problems.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

Vocabulary
Concepts
Skills

1.01 Write equivalent forms of algebraic expressions to solve problems.

Vocabulary
Concepts
Skills

1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.

Vocabulary
Concepts
Skills

Constant

Coefficient

Ratio

Proportion

1.03 Model and solve problems using direct variation.

A. For the years 1980-88, the function $v(x) = 6.25x$ represents the number of VCRs owned (in millions). For years 1989-94, $w(x) = 3.2x + 29.2$ represents the number of VCRs owned (in millions). In both functions, x represents the number of years since 1980. Discuss the changes in growth of VCR ownership described in the models.

B. In 1990, US exports by North Carolina agriculture and industries were worth $8.01 billion. By 1997, exports from North Carolina had increased to $16.402 billion. Assuming the growth in exports has been constant, what is the average annual growth in exports for the period?

C. The relative values of various currencies change frequently. On February 3, 2003, the British pound had a value of $1.6475 and the Swiss franc had a value of $0.7345. How many Swiss francs are equivalent to one British pound?

D. Amusement and theme parks in the United States had a total payroll of $1.69 billion last year. The average annual salary of park employees was $15,258 that year. How many people were employed at amusement and theme parks last year?

E. The cost of living indices measure relative price levels for consumer goods and services. In Asheville the index for transportation costs is 107.1. This means transportation costs in Asheville are 107.1% of the national average. Nationally, the weekly cost of food for a family of four is $165.30. The indices for food costs in Greenville and Wilmington are 94.7 and 107.2, respectively. On average, how much more does a family spend for food in Wilmington compared with a family in Greenville?

F. The period of a simple pendulum varies directly as the square root of its length. If a pendulum three feet long has a period of 4.8 seconds, find the period of a pendulum half as long.

Vocabulary
Concepts
Skills

Ordered Array

Dimensions

Row

Column

Element

Identity

Inverse

Addition

Subtraction

Multiplication

Determinant

Unit Matrix

90º Rotations

Translation

Reflection

Dilation

Composition

1.04 Operate with matrices to model and solve problems.

A. Given;

$$\mathbf{A} = \begin{bmatrix} 2 & 6 & 7 \\ 4 & -1 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 2 \\ 6 & 1 \\ 4 & 5 \\ 0 & -3 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 4 & 4 \\ 0 & 3 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 5 & 8 & -4 \\ 4 & 7 & -3 \end{bmatrix}$$

Without a calculator, find A + C; 4B; 3C - 2A; C - B; 2B + 5D.
Justify each result.

B. The matrices below show trade between the United States and its five largest trading partners. For the period shown, find the total value of goods exchanged annually between the United States and its trading partners. Find the balance of trade annually between the United States and its trading partners. Expand the matrices to include the most recent data possible and revisit total value and balance of trade. Discuss any patterns that may be apparent in the resulting matrices.

Exports to ... (billions of dollars)

	1994	1995	1996	1997	1998
Canada	114.4	127.2	134.2	151.8	156.3
China	9.3	11.8	12.0	12.9	14.3
Germany	19.2	22.4	23.5	24.5	26.6
Japan	53.5	64.3	67.6	65.5	57.9
Mexico	50.8	46.3	56.8	71.4	79.0

Imports from ... (billions of dollars)

	1994	1995	1996	1997	1998
Canada	128.4	144.4	155.9	168.2	174.8
China	36.8	45.5	51.5	62.6	71.2
Germany	31.7	36.8	38.9	43.1	49.8
Japan	119.2	123.5	115.2	121.7	122.0
Mexico	49.5	62.1	74.3	85.9	94.7

Vocabulary Concepts Skills

C. $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$ represents ΔABC using its vertices. Transformations of ΔABC are described in each expression. Evaluate each expression and describe ΔA′B′C′ with respect to ΔABC.

(1.) $2 \bullet \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$

(2.) $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 5 & 5 & 5 \end{bmatrix}$

(3.) $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & 1.5 \end{bmatrix} \bullet \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(4.) $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$

(5.) $\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$

(6.) $\begin{bmatrix} 1.5 & 0 \\ 0 & 2.5 \end{bmatrix} \bullet \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ 0 & 7 \end{bmatrix} \bullet \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

D. The coordinates for ΔMAT are M(-6, 8), A(3, 5), and T(-1, -4). Write a matrix expression that would rotate ΔMAT 90^{o} counterclockwise.

E. The coordinates for quadrilateral MNOP are M(-1, 3), N(-5, -1), O(-1, -2), and P(3, 2). Write a matrix expression that will shift MNOP six units left and four units down. What are the coordinates for O′? MNOP is dilated by a facor of 1.4. Write the matrix that represents M′N′O′P′.

F. What is the matrix expression that will reflect ΔABC, $\begin{bmatrix} 2 & -3 & 1 \\ 4 & 5 & -5 \end{bmatrix}$, over the x-axis? What is the matrix expression that will reflect ΔABC over the y-axis and locate B′ at (0, 0)? Write the matrix expression that dilates ΔABC horizontally by a factor of four and vertically by a factor of three. Write the matrix expression that dilates ΔABC by a factor of two and locates A′ at (9, 3).

G. Write the matrix expression that translates ΔABC, $\begin{bmatrix} 4 & -7 & 2 \\ 1 & 1 & -1 \end{bmatrix}$, so that B′ is at (-6, -2).

Vocabulary
Concepts
Skills

Pythagorean Theorem

Distance Formula

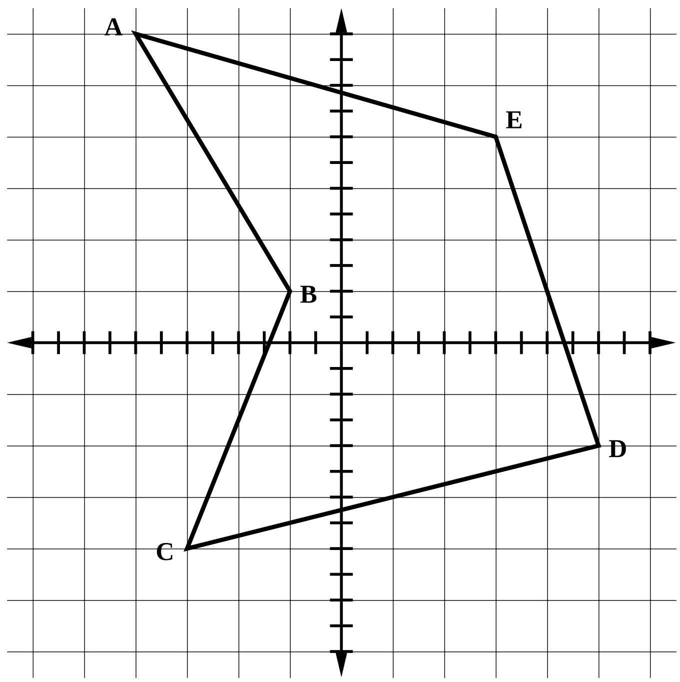
Midpoint

2.01 Find the lengths and midpoints of segments to solve problems.

A. Find the perimeter of the triangle with vertices (4,0), (-2,-2), and (2,6).

B. Parallelogram ABCD has vertices (8, 9), (9, 3), (2, 5), and (1, 11). What are the coordinates of the intersection of the diagonals? What is the perimeter of ABCD?

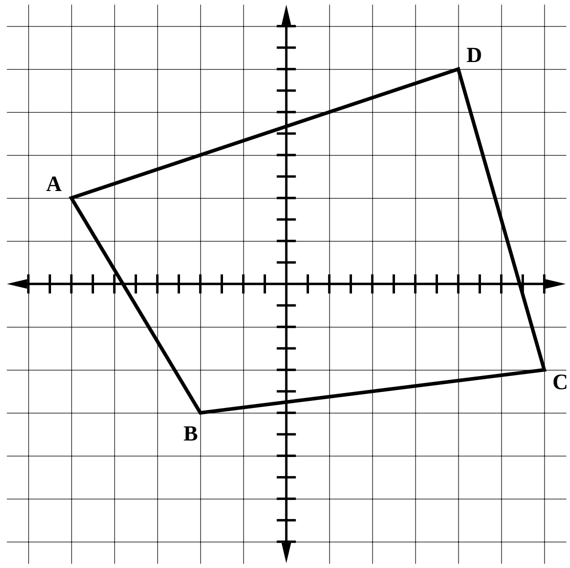
C. Find the perimeter of ABCDE.



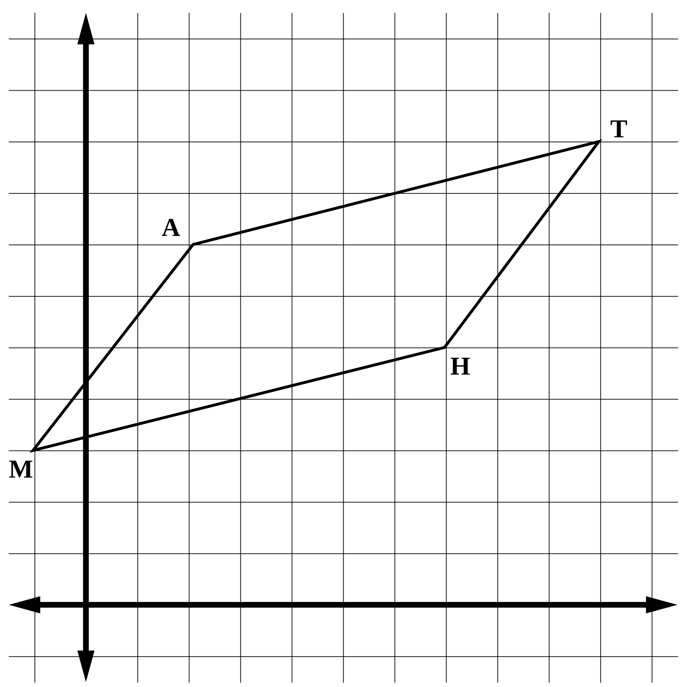
Vocabulary
Concepts
Skills

2.02 Use the parallelism or perpendicularity of lines and segments to solve problems.

A. Connect the midpoints of the sides of ABCD consecutively to form a new quadrilateral. Which special quadrilateral is it? Justify.



B. Give the equation of the line that includes an altitude of parallelogram MATH.



C. ΔABC is a right triangle with vertices A (-3, 3) and B (2, 2) and $m\angle A = 90$. What is the equation for $\overleftrightarrow{AC}$?

Vocabulary Concepts Skills

Right Triangle

Hypotenuse

Legs

Altitude

Sine

Cosine

Tangent

Arcsine

Arccosine

Arctangent

45^o-45^o-90^o Triangle

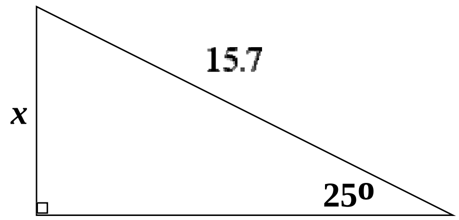
30^o-60^o-90^o Triangle

Angle of Elevation

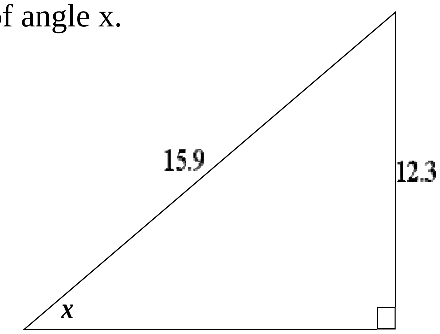
Angle of Depression

2.03 Use the trigonometric ratios to model and solve problems.

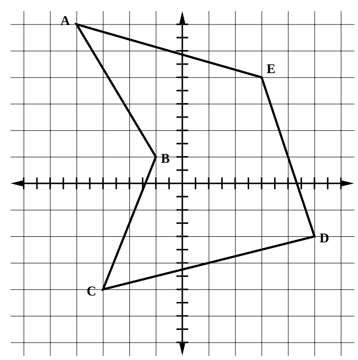
A. Find x.



B. Find the measure of angle x.



C. Find the measures of interior ∠CDE and exterior ∠ABC.



D. From the top of a building 50 feet high the angles of elevation and depression of the top and bottom of another building are 19.7^o and 26.6^o, respectively. What are the height and distance of the second building?

E. At two tracking stations ten miles apart, the elevation angles of a passing airliner are 16.5^o and 38.3^o, respectively. At what altitude is the airliner flying?

F. As a balloon passes between two points and 2 miles apart, the angles of elevation of the balloon at these points are 27.3^o and 41.8^o, respectively. Find the altitude of the balloon.

G. The top of a lighthouse is 230 feet above the sea. How far away is an object which is just “on the horizon”? (Assume the earth is a sphere of radius 3956 miles.) What must be the elevation of an observer in order that she may be able to see an object on the earth thirty miles away?

Vocabulary
Concepts
Skills

90° Rotations

Translation

Reflection

Dilation

Mapping

Isometry

Clockwise

Counterclockwise

Pre-image

Image

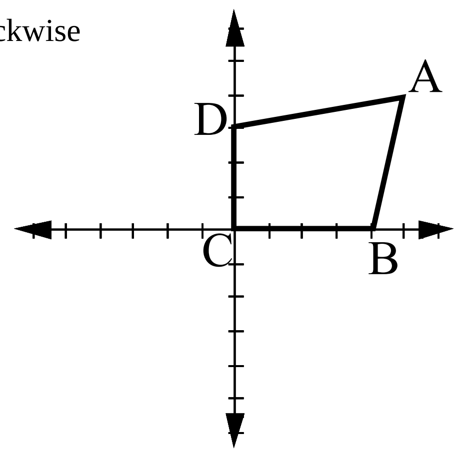
Composition

$(x', y') =$
$(ax + b, cy + d)$

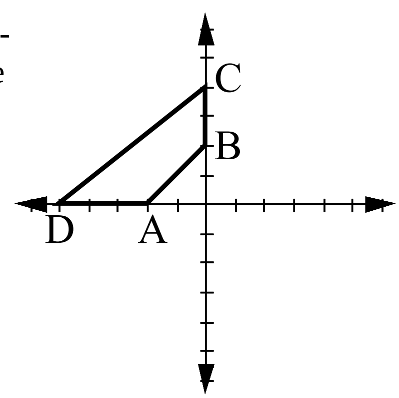
2.04 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

A. If ΔA'C'B' was translated by $(x', y') = (x + 2, y - 6)$ and the coordinates of ΔA'C'B' are A'(-8, 9), C'(7, -3), and D'(2, 6), what were the coordinates of the pre-image?

B. ABCD is rotated 270° clockwise about the origin. Describe the transformation algebraically.



C. ABCD is reflected across the x-axis and translated five units to the left. Describe the transformation algebraically.



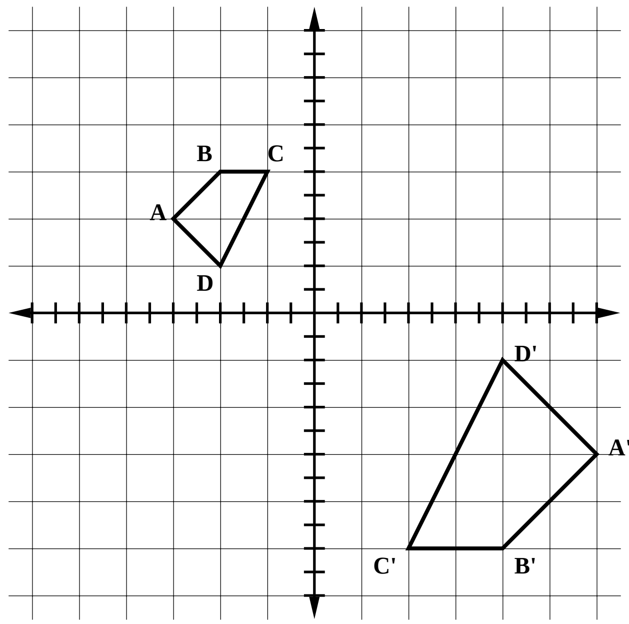
D. ΔABC, with vertices A(2, 8), B(5, 3), and C(6, 8) is transformed according to $(x', y') = (-2x + 3, y - 4)$. Graph ΔABC and ΔA´B´C´; describe the transformation.

E. Transform ΔRST, with vertices R(2, 2), S(3, 6), and T(8, 3), so that its linear dimensions double, but vertex S´ is located at (3, 6). Describe the transformation algebraically.

Vocabulary Concepts Skills

F. ΔABC has vertices A(9, 6), B(12, 3), and C(6, -1). ΔPQR has vertices P(1, 6), Q(-2, 3), and R(4, -1). If ΔPQR is the reflected image of ΔABC, what is the equation of the line of reflection? Write the algebraic expression that represents the transformation.

G. Algebraically describe the transformation of ABCD to A′B′C′D′.



Vocabulary
Concepts
Skills

3.01 Describe data to solve problems.

Vocabulary
Concepts
Skills

3.02 Create and use, for sets of data, calculator-generated models of linear, exponential, and quadratic functions to solve problems.

A. Various departments of state and federal government use population projections to plan. Construct an algebraic model of North Carolina's population growth using 1790-1990 data, check it for validity (goodness-of-fit, accuracy with respect to the 2000 census), adjust the model to accommodate the 2000 census, and use the model to estimate North Carolina's 2010 census.

1790	393,751	1860	992,622	1930	3,170,276
1800	478,103	1870	1,071,361	1940	3,571,623
1810	555,500	1880	1,399,750	1950	4,061,929
1820	638,829	1890	1,617,949	1960	4,556,155
1830	737,987	1900	1,893,810	1970	5,084,411
1840	753,419	1910	2,206,287	1980	5,880,095
1850	869,039	1920	2,559,123	1990	6,628,637

B. The table below shows the price current owners of a particular luxury car are asking in their advertisements in the classified section of the newspaper. Based on this information, determine a function that models the depreciation (loss in value) of the car.

1999	28,900	1994	11,995
2002	49,900	1993	11,800
1996	14,900	1999	32,900
1998	24,995	1999	28,900
1993	11,750	1998	19,900
1998	22,900	1997	16,750

Vocabulary Concepts Skills

C. Graph and describe the newspaper circulation (in millions) data shown. What variables affect newspaper circulation? Create an algebraic model of the data (let x = 20 for 1920). According to the model, will the newspaper circulation drop below 50 million? If it does, when?

1920	27.8	1960	58.9	1992	60.2
1925	33.7	1965	60.4	1993	59.8
1930	39.6	1970	62.1	1994	59.3
1935	38.2	1975	60.7	1995	58.2
1940	41.1	1980	62.2	1996	57.0
1945	48.4	1985	62.8	1997	56.7
1950	53.8	1990	62.3	1998	56.2
1955	56.1	1991	60.7		

Vocabulary
Concepts
Skills

Graph

Intersection

Domain

Range

Parallel

Substitution

Elimination

Solution

4.01 Use systems of linear equations or inequalities in two variables to model problems. Solve using tables, graphs, and algebraic properties; justify steps used.

A. The bill for a lunch of three hamburgers and two drinks is $9.67. The bill for a lunch of four hamburgers and three drinks is $13.21. What is the total cost of one hamburger and one drink?

B. For a special order, the Coverup Company manufactured 1200 shirts. Sweatshirts were priced at $14 each and T-shirts at $8 each. The company received a total of $11,400 for the shirts. How many of each type of shirt did the Coverup Company manufacture for this order?

C. A movie theater charges $7 for an adult's ticket and $4.50 for a child's ticket. On a recent night, the sale of child's tickets was three times the sale of adult's tickets. If the total amount collected for ticket sales was $2,009, how many adults purchased tickets?

D. Solve exactly without a calculator; show and justify each step.

$$2x + 3y = 12$$
$$6x - 5y = -24$$

E. Transportation costs for travel between selected cities are shown below. Assume that the costs identified represent a linear trend. Determine the distance at which driving a car is less expensive than riding the train. When does it become cheaper to fly rather than drive? For what distance is the train the most expensive mode of travel? Identify some advantages and disadvantages for each mode of transportation. What other variables affect the cost of travel?

	Distance	Car	Air	Rail
Raleigh – Charlotte	300	$108	$224	$40
Raleigh – NewYork	1300	$468	$169	$147

Vocabulary Concepts Skills

Graph

Parabola

Vertex

Domain

Range

Independent

Dependent

Intercepts

Solutions

Zeros

Roots

Coefficients

Increasing

Decreasing

Maximum

Minimum

Evaluate

Factor

Properties of Equality

Distributive Property

$f(x) = ax^2 + bx + c$

4.02 Use quadratic functions to model and solve problems; justify results.

A. The function $g(x) = -0.005x^2 + 0.12x + 1.179$ describes the monthly price of gasoline for a recent 18-month period. At what month did the prices reach their peak? How long were the prices above $1.50? Assuming the function continues to model gasoline prices, how long will it be until the price returns to its initial value of $1.179 per gallon?

B. The function $c(x) = 0.194x^2 - 2.12x + 18$ models the value of a share of stock in a computer camera company for a recent 12-month period. What was the lowest price of the camera stock? What was the greatest price for the 12-month period? If the function continues to accurately model the value of the stock, will the stock ever double its initial value of $18? When?

C. Find the exact values of x that satisfy $7(x^2 - 7x - 7) = 67x + 2$.

D. Find the exact values of x that satisfy $4x^2 + 3x - 9 = 6 - x + x^2$.

E. Without a calculator, identify similarities and differences among $y = x^2$, $y = x^2 + 5$, and $y = x^2 - 3$.

F. Without a calculator, identify similarities and differences among $y = x^2$, $y = 2.5x^2$, $y = -1.5x^2$, $y = 0.3x^2$, and $y = -3x^2$.

G. The function $f(x) = -0.019x^2 + 3.04x - 58.87$ describes newspaper circulation (millions) in the United States for 1920-98 (x = 20 for 1920). Identify periods of increasing and decreasing circulation. According to the function, when did newspaper circulation peak? When will circulation approximate 45 million?

Vocabulary
Concepts
Skills

4.03 Use power models to solve problems.

A. The function $\mathbf{p(x) = ax^{-0.9}}$ generates the prize money awarded to the top 15 finishers (**x**) in any Grand Prix automobile race (**a** is the amount awarded for first place). How much is a tenth place finisher awarded if first place receives $95,000? What is the first place prize money if eighth place receives $37,242?

B. The function $\mathbf{g(x) = ax^{-0.82}}$ generates the prize money awarded to the top 25 finishers (**x**) in any of the tournaments on the women's golf tour (**a** is the amount awarded for first place). How much is a 12^{th} place finisher awarded if first place receives $110,000? What is the first place prize money if seventh place receives $23,887?