

Updated 04/27/05

# Indicators for K-12 Mathematics: Geometry

*Geometry* continues students' study of geometric concepts building upon middle school topics. Students will move from an inductive approach to deductive methods of proof in their study of geometric figures. Two- and three-dimensional reasoning skills will be emphasized and students will broaden their use of the coordinate plane. Appropriate technology, from manipulatives to calculators and graphics software, should be used regularly for instruction and assessment.

## Prerequisites

- *Apply geometric properties and relationships to solve problems.*
- *Use formulas to solve problems.*
- *Define and use linear expressions to model and solve problems.*
- *Operate with matrices to model and solve problems.*

## **STATE BOARD OF EDUCATION**

### **HOWARD N. LEE**

Chairman  
Raleigh

### **JANE P. NORWOOD**

Vice Chair  
Charlotte

### **KATHY A. TAFT**

Greenville

### **MICHELLE HOWARD-VITAL**

Wilmington

### **EDGAR D. MURPHY**

Durham

### **EVELYN B. MONROE**

West End

### **MARIA T. PALMER**

Chapel Hill

### **ROBERT "TOM" SPEED**

Boone

### **WAYNE MCDEVITT**

Asheville

### **JOHN TATE III**

Charlotte

### **BEVERLY PERDUE**

Lieutenant Governor  
New Bern

### **RICHARD MOORE**

State Treasurer  
Kittrell

## **NC DEPARTMENT OF PUBLIC INSTRUCTION**

**Patricia N. Willoughby, State Superintendent**

**301 N. Wilmington Street :: Raleigh, North Carolina 27601-2825 :: [www.ncpublicschools.org](http://www.ncpublicschools.org)**

In compliance with federal law, NC Public Schools administers all state-operated educational programs, employment activities and admissions without discrimination because of race, religion, national or ethnic origin, color, age, military service, disability, or gender, except where exemption is appropriate and allowed by law.

#### **Inquiries or complaints should be directed to:**

Dr. Elsie C. Leak, Associate Superintendent :: Office of Curriculum and School Reform Services  
6307 Mail Service Center :: Raleigh, NC 27699-6307 :: Telephone 919-807-3761 :: Fax 919-807-3767  
Visit us on the Web: [www.ncpublicschools.org](http://www.ncpublicschools.org)

Updated 04/27/05

# Geometry

**GOAL 1: The learner will perform operations with real numbers to solve problems.**

- 1.01 Use the trigonometric ratios to model and solve problems involving right triangles.
- 1.02 Use length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.
- 1.03 Use length, area, and volume to model and solve problems involving probability.

**GOAL 2: The learner will use geometric and algebraic properties of figures to solve problems and write proofs.**

- 2.01 Use logic and deductive reasoning to draw conclusions and solve problems.
- 2.02 Apply properties, definitions, and theorems of angles and lines to solve problems and write proofs.
- 2.03 Apply properties, definitions, and theorems of two-dimensional figures to solve problems and write proofs:
  - a) Triangles.
  - b) Quadrilaterals.
  - c) Other polygons.
  - d) Circles.
- 2.04 Develop and apply properties of solids to solve problems.

**GOAL 3: The learner will transform geometric figures in the coordinate plane algebraically.**

- 3.01 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.
- 3.02 Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

**Vocabulary  
Concepts  
Skills**

Right  
Triangle

Hypotenuse

Legs

Altitude

Sin A

Cos A

Tan A

Sin<sup>-1</sup> A  
(Arcsine)

Cos<sup>-1</sup> A  
(Arccosine)

Tan<sup>-1</sup> A  
(Arctangent)

45°-45°-90°  
Triangle

30°-60°-90°  
Triangle

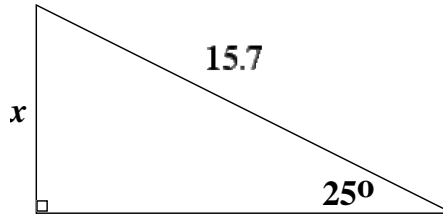
Angle  
of  
Elevation

Angle  
of  
Depression

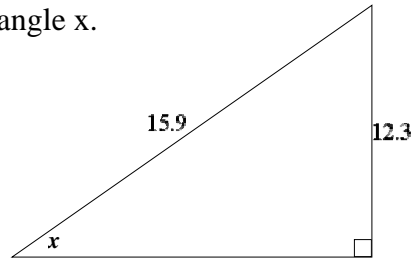
Simplify Irrational  
Expressions

# 1.01 Use the trigonometric ratios to model and solve problems involving right triangles.

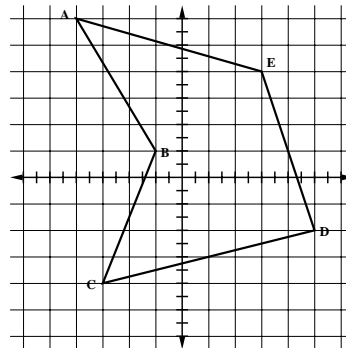
A. Find  $x$ .



B. Find the measure of angle  $x$ .



C. Find the measures of interior  $\angle CDE$  and exterior  $\angle ABC$ .



D. From the top of a building 50 feet high, the angles of elevation and depression of the top and bottom of another building are  $19.7^\circ$  and  $26.6^\circ$ , respectively. What is the height of the second building and how far away is it?

E. At two tracking stations ten miles apart, the elevation angles of a passing airliner are  $16.5^\circ$  and  $38.3^\circ$ , respectively. At what altitude is the airliner flying?

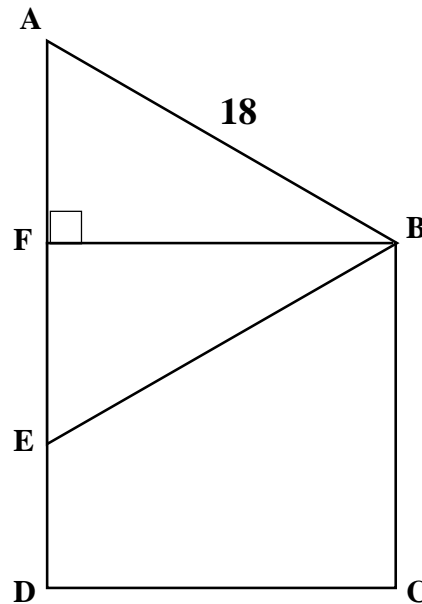
F. As a balloon passes between two points, two miles apart, the angles of elevation of the balloon at these points are  $27.3^\circ$  and  $41.8^\circ$ , respectively. Find the altitude of the balloon.

G. The top of a lighthouse is 230 feet above the sea. How far away is an object which is just “on the horizon”? (Assume the earth is a sphere of radius 3956 miles.) What must be the elevation of an observer in order that she can see an object on the earth thirty miles away?

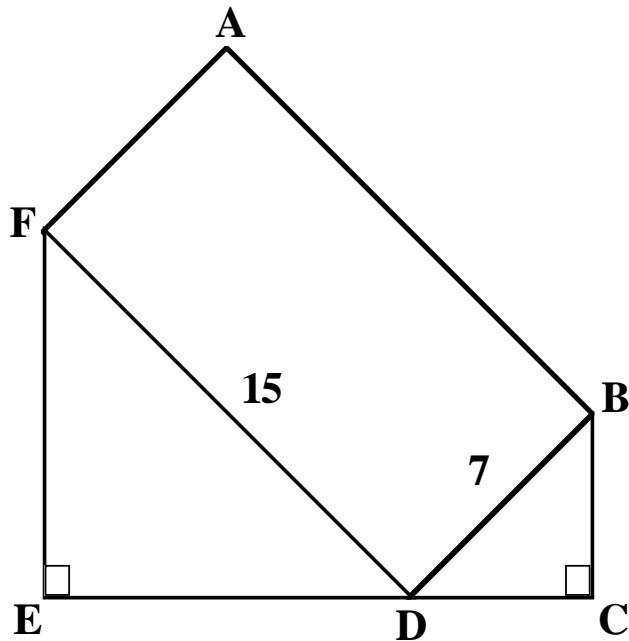
Updated 04/27/05

Vocabulary  
Concepts  
Skills

H. ABE is an equilateral triangle and BCDF is a square. What is the perimeter of BCDE? What is the area of BCDE?



I. ABDF is a rectangle and both  $\triangle DEF$  and  $\triangle BCD$  are isosceles. What is the perimeter of ABCEF? What is the area of ABCEF?



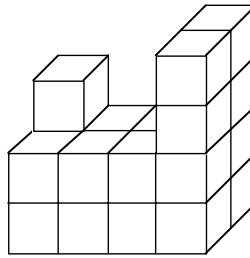
*Vocabulary  
Concepts  
Skills*

- Perimeter
- Circumference
- Lateral Area
- Surface Area
- Apothem
- Slant Height
- Great Circle
- Minor Arc
- Major Arc
- Height
- Altitude
- Irregular
- Composite
- Truncated
- Oblique

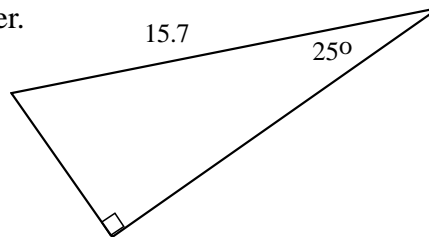
# 1.02 Use length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.

A. A chimney (cylindrical) has an outside radius of 6.5 inches and an inside radius of 5.3 inches. The chimney is six feet long. To the nearest tenth, how much surface is exposed?

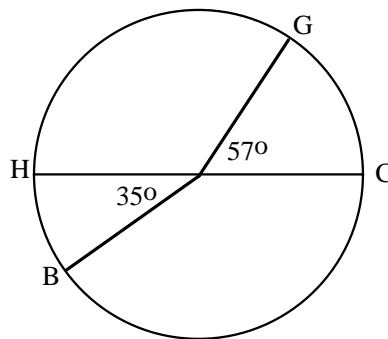
B. The figure shown was built with cubes. The front edge of the base is 14 cm long. What is the volume of the figure? What is its surface area?



C. Find the perimeter.



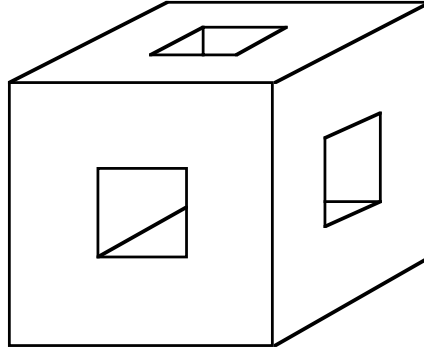
D. The circle is divided as shown with diameter  $\overline{HC}$ , two central angles indicated, and an area of  $414 \text{ in}^2$ . Find the length of minor arc  $\widehat{BG}$ .



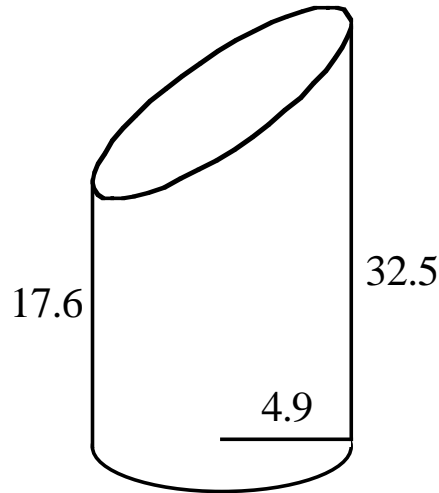
**Vocabulary**  
**Concepts**  
**Skills**

- Sector
- Arc
- Prism
- Pyramid
- Cylinder
- Cone
- Sphere
- Cube
- Faces
- Vertices
- Base
- Edge
- 3-D Coordinates
- Distance Formula (3-D)
- Midpoint (3-D)

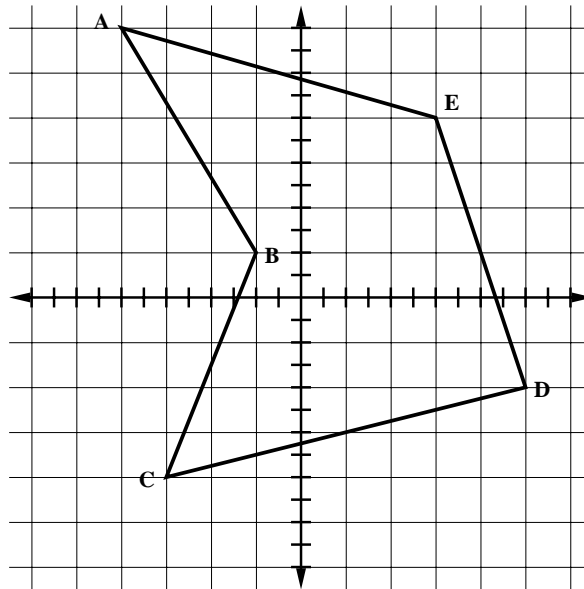
E. The plastic cube shown originally had a volume of  $2500 \text{ cm}^3$ . The front face is drawn to proportion. Square holes were cut through to the opposite faces. How much surface is exposed?



F. A glass cylinder is cut as shown. Find the volume and total surface area. The area of the ellipse is  $\pi ab$  where  $a$  is the length of the semi-major axis (half the length of the diagonal cut) and  $b$  is the length of the semi-minor axis (half the length of the radius).



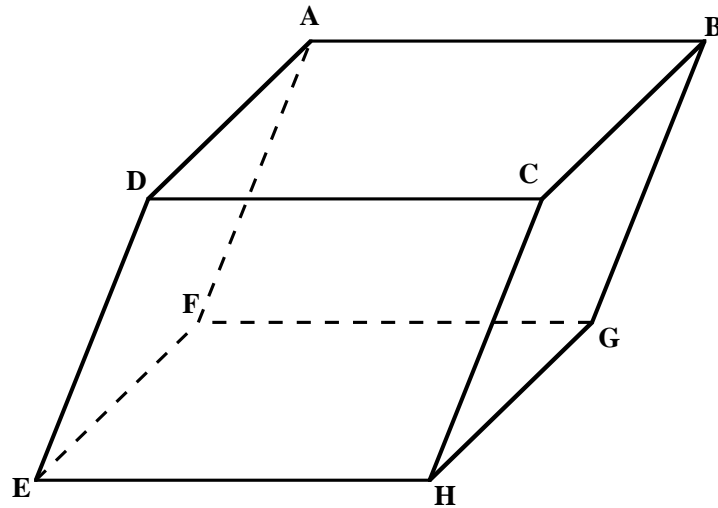
G. Find the exact area and perimeter of ABCDE.



Updated 04/27/05

Vocabulary  
Concepts  
Skills

H. Find the center of prism with vertices  $A(3, 10, 8)$ ,  $B(10, 10, 8)$ ,  $C(10, 6, 8)$ ,  $D(3, 6, 8)$ ,  $E(1, 6, 3)$ ,  $F(1, 10, 3)$ ,  $G(8, 10, 3)$ , and  $H(8, 6, 3)$ .

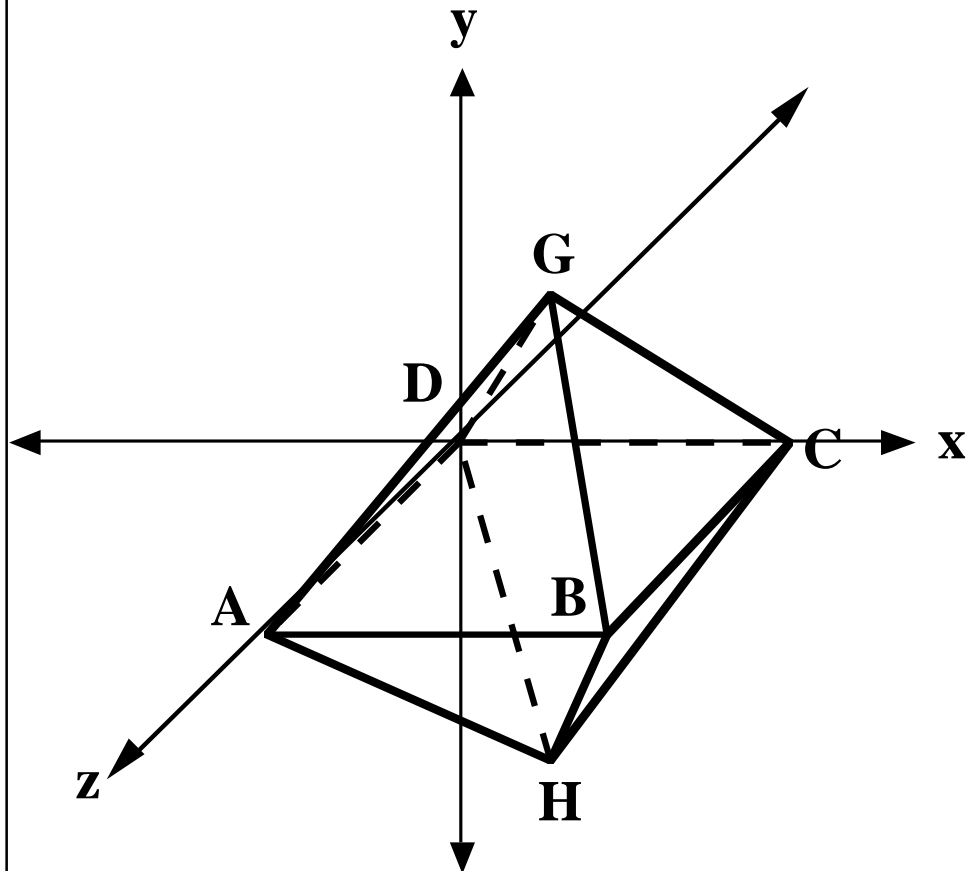


I.  $GABCDH$  is a regular octahedron with vertices  $G(a, b, c)$ ,  $A(0, 0, 5)$ ,  $B(5, 0, 5)$ ,  $C(5, 0, 0)$ , and  $D(0, 0, 0)$ .

What is the ordered triple for  $H$ ?

What is the volume of  $GABCDH$ ?

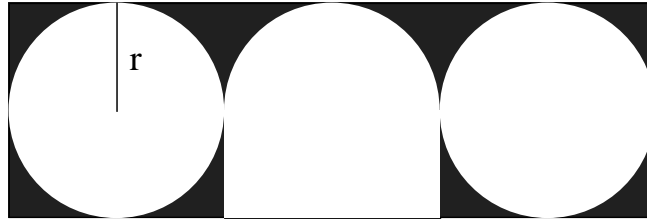
What is the surface area of  $GABCDH$ ?





# 1.03 Use length, area, and volume to model and solve problems involving probability.

A. If a dart hits the rectangle, find the probability of hitting the shaded area in terms of  $r$ .

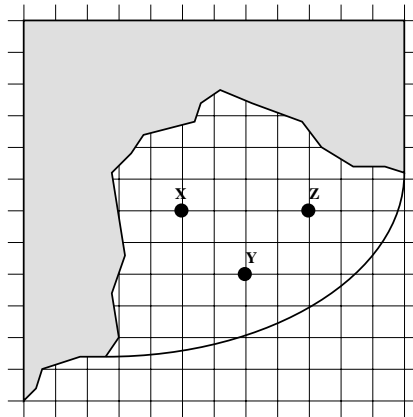


B. Catawba Bay (1 unit = 1 km) covers an area of about  $35 \text{ km}^2$  and has an average depth of 20 m. Microphones are placed on the floor of the bay at the locations indicated (X, Y, and Z) to detect dolphins which come to feed. The locations have the following depths: X, 10 m; Y, 35 m; and Z, 12 m.

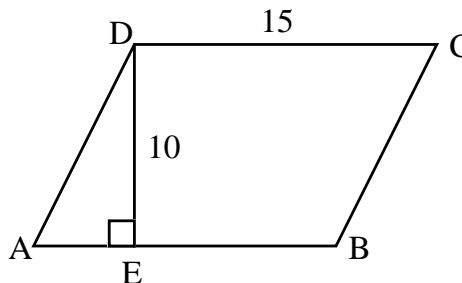
The microphones have a range of 800 m. What are the chances that the microphones will detect a dolphin at any time while it is in the bay? The

volume of the zone of a sphere is given by  $V = \frac{1}{6}\pi h(3a^2 + 3b^2 + h^2)$

where  $h$  is the height of the zone and  $a$  and  $b$  are radii of the respective bases.



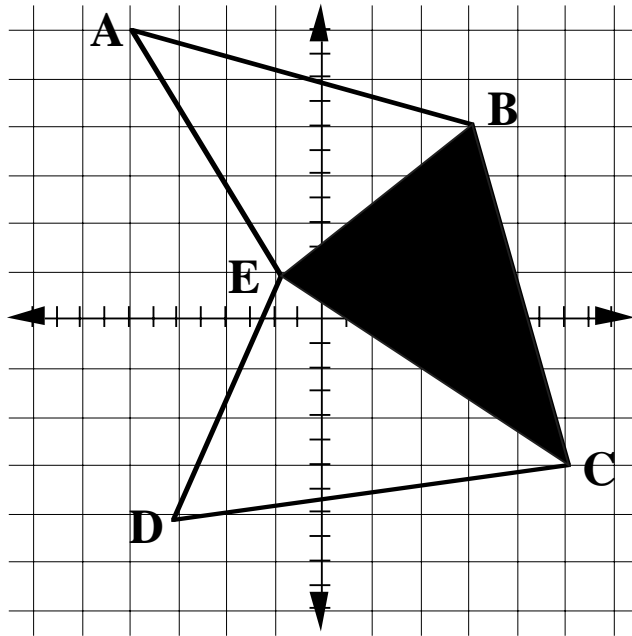
C. In parallelogram ABCD,  $DC = 15 \text{ m}$  and  $DE = 10 \text{ m}$ . If F is a point randomly selected from the interior of ABCD, what is the probability that the area of  $\triangle AFB$  is less than  $30 \text{ m}^2$ ?



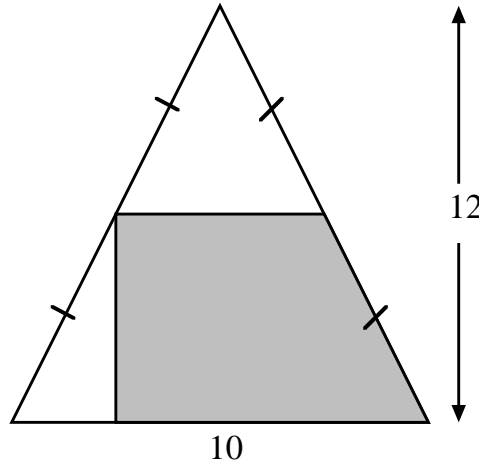
Updated 04/27/05

Vocabulary  
Concepts  
Skills

D. If a point is in the interior of ABCDE, find the probability of it being in the shaded area.

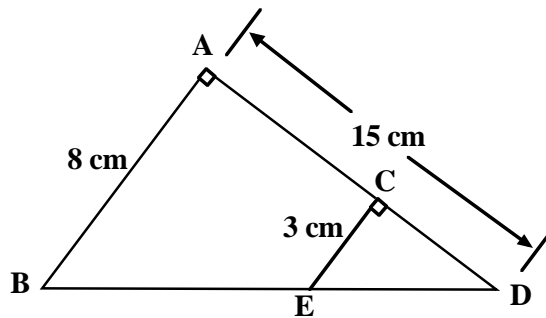


E. In the isosceles triangle shown, find the probability of hitting the shaded area if you hit the interior of the triangle.



F. If a point is selected at random in the interior of a circle, find the probability that the point is closer to the edge of the circle than the center.

G. Suppose a point is picked at random in  $\triangle ABD$ . What is the probability that the point is outside  $\triangle CED$ ?



*Vocabulary  
Concepts  
Skills*

Conditional

Converse

Inverse

Negation

Contrapositive

Biconditional

Logic

Theorems

Properties

Postulates

Definitions

## 2.01 Use logic and deductive reasoning to draw conclusions and solve problems.

A. If Shane is an athlete and he is salaried, then Shane is a professional. Shane is not a professional. Shane is an athlete. Which statement must be true?

- a) Shane is an athlete and he is salaried.
- b) Shane is a professional or he is salaried.
- c) Shane is not salaried.
- d) Shane is not an athlete.

B. When the statement "If A, then B" is true, which statement must also be true?

- (a) If B, then A.
- (b) If not A, then B.
- (c) If not B, then A.
- (d) If not B, then not A.

C. If two triangles are congruent, then all of the corresponding parts are congruent. Write the converse; the contrapositive; the inverse.

D. If Sue goes out on Friday night and not on Saturday night, then she does not study. If Sue does not fail mathematics, then she studies. Sue does not fail mathematics. If Sue does not go out on Friday night, then she watches a movie. Sue does not watch a movie.

Prove: Sue goes out on Saturday night.

E. Which is the converse of the statement "If today is Thanksgiving, then there is no school"?

- a) If there is school, then today is not Thanksgiving.
- b) If there is no school, then today is Thanksgiving.
- c) If today is Thanksgiving, then there is school.
- d) If today is not Thanksgiving, then there is school.

F. If I receive a check for \$500, then we will go on a trip. If the car breaks down, then we will not go on the trip. Either I receive a check for \$500 or we will not buy souvenirs. The car breaks down.

Prove: We will not buy souvenirs.

*Vocabulary  
Concepts  
Skills*

## 2.02 Apply properties, definitions, and theorems of angles and lines to solve problems and write proofs.

Adjacent Angles

Vertical Angles

Linear Pair

Complementary

Supplementary

Alternate Interior

Corresponding

Same Side Interior

Transversal

Angle  
Addition  
Postulate

Midpoint

Segment Bisector

Angle Bisector

Perpendicular  
Bisector

Parallel

Perpendicular

Skew

Collinear

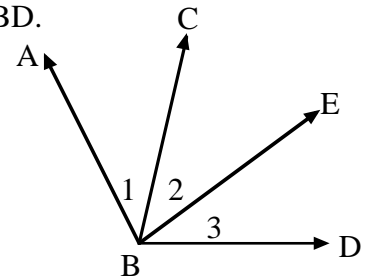
Slope

Length

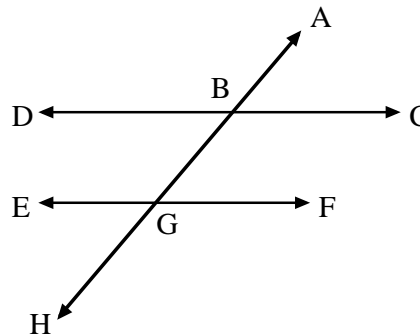
Forms of Proof

A. If  $\overrightarrow{SU}$  bisects  $\angle RST$ ,  $m\angle RSU = (2x - 11)$ , and  $m\angle RST = (3x + 23)$ , find  $m\angle TSU$ . Use a two-column format to show your work.

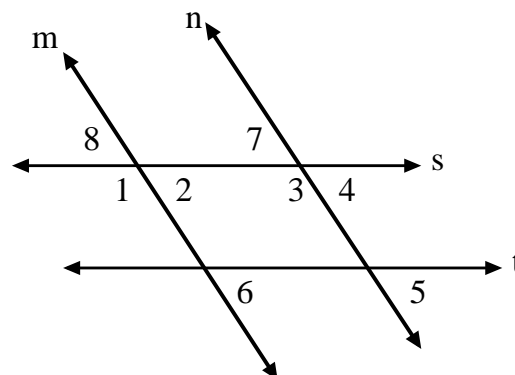
B. Write a flow diagram to illustrate the following proof.  
Given  $\angle ABE \cong \angle CBD$ , prove  $\angle ABC \cong \angle EBD$ .



C.  $\overleftrightarrow{DC}$  and  $\overleftrightarrow{EF}$  are parallel,  $m\angle EGH = (2x - 5)$ , and  $m\angle GBC = (3x - 10)$ . Determine the  $m\angle ABC$ . Explain your reasoning.



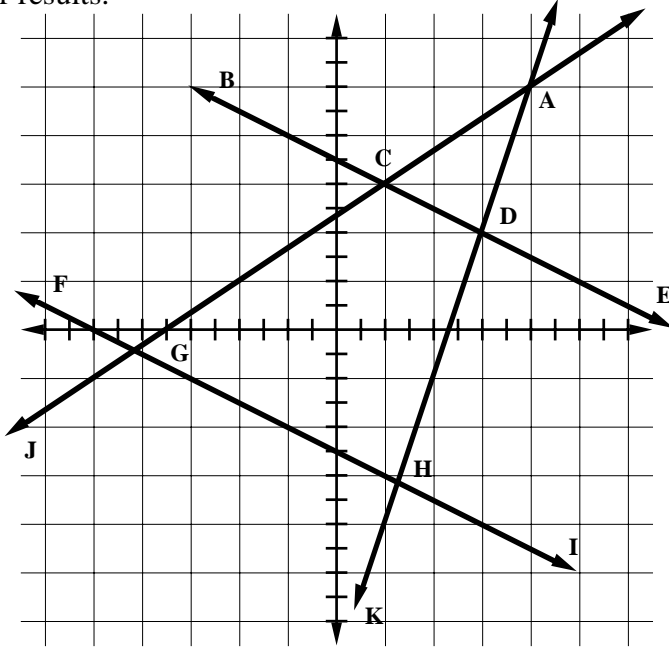
D. Given  $\angle 1 \cong \angle 3$  and  $\angle 7 \cong \angle 6$ , prove  $\overleftrightarrow{s}$  and  $\overleftrightarrow{t}$  are parallel.



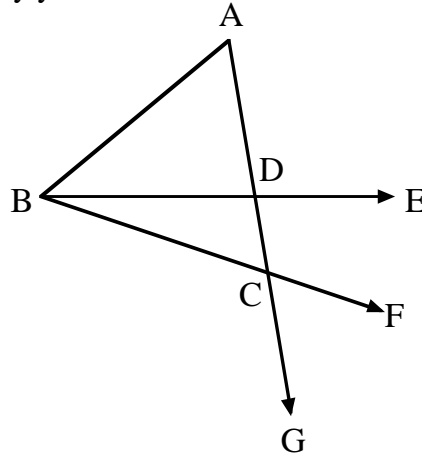
Updated 04/27/05

Vocabulary  
Concepts  
Skills

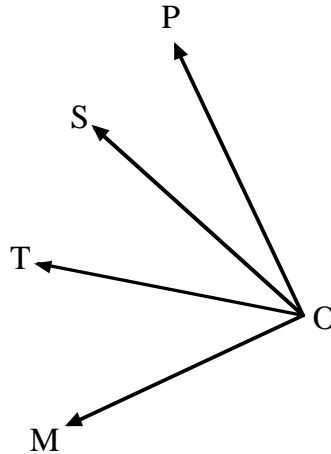
E. Find  $m\angle FGJ$ ,  $m\angle KHI$ , and  $m\angle CAD$  to the nearest hundredth. Justify your results.



F. Find  $m\angle DBC$  if  $m\angle ACB = (21x + 36)$ ,  $m\angle FCG = (13x + 42)$ , and  $m\angle ADB = 83$ . Justify your results.



G. Given  $\overrightarrow{OS}$  bisects  $\angle TOP$ ,  $\overrightarrow{OM} \perp \overrightarrow{OP}$ ,  $m\angle MOT = (3x + 3)$ , and  $m\angle TOS = (2x + 5)$ , find  $m\angle TOP$ . Justify your results.



Vocabulary  
Concepts  
Skills

Interior  
Angle

Exterior  
Angle

Congruence

Equality

SAS

SSS

ASA

HL

AAS

CPCTC

AA

Scale  
Factor

Geometric  
Mean

Pythagorean  
Theorem

Opposite

Adjacent

Included Angle

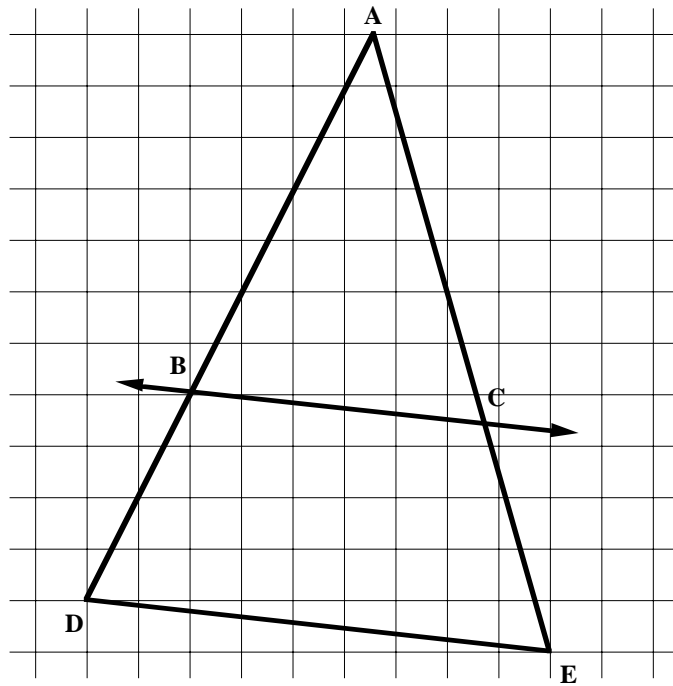
Proportional

Forms of Proof

## 2.03 Apply properties, definitions, and theorems of plane figures to solve problems and write proofs.

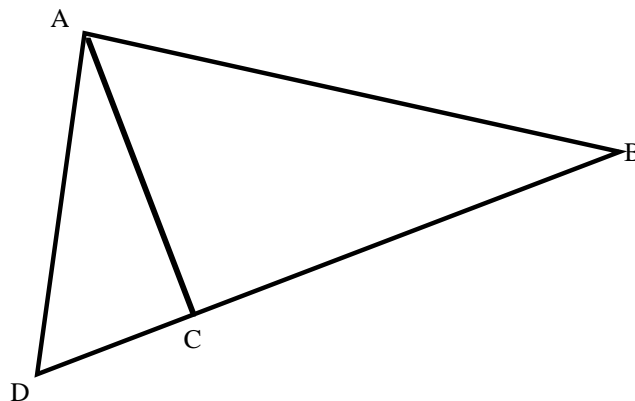
### a) Triangles

A.  $\overleftrightarrow{BC}$  and  $\overline{DE}$  are parallel. Find the perimeters of  $\triangle ABC$  and  $BCED$ . Justify your results.



B. In  $\triangle ABC$ ,  $AB = 27$  and  $BC = 15$ . What is true about the length of  $\overline{AC}$ ? Explain.

C. In  $\triangle ADB$ ,  $m\angle DAB = m\angle ACB = 90$ ,  $AB = 19$ , and  $AC = 9$ . Find  $DC$ . Justify your results.

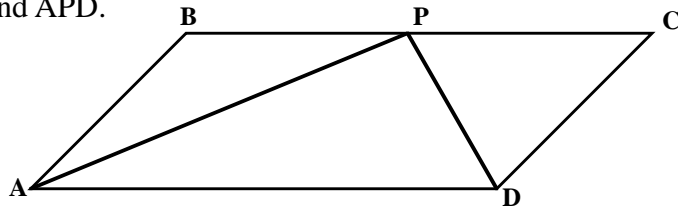


Updated 04/27/05

**Vocabulary**  
**Concepts**  
**Skills**

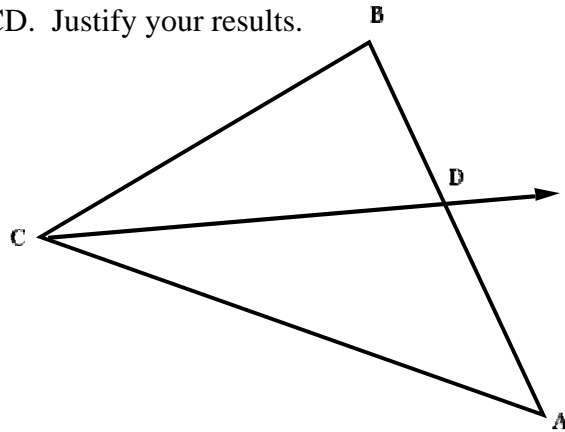
- Scalene
- Isosceles
- Equilateral
- Equiangular
- Right
- Acute
- Obtuse
- Altitude
- Median
- Perpendicular
- Bisector
- Angle Bisector
- Hypotenuse
- Legs
- Midsegment

D. In parallelogram ABCD, the bisectors of two consecutive angles (A and D) meet at a point P on a non-adjacent side. Describe triangles ABP, PCD, and APD.

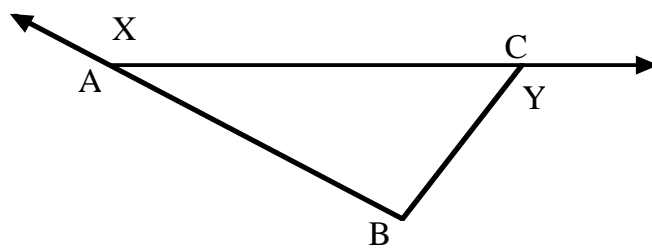


E.  $\triangle ABC$  is equilateral with vertices A (5, 3) and B (10, 8). Locate vertex C. Explain.

F.  $\overrightarrow{CD}$  is the bisector of  $\angle ACB$ ,  $m\angle A = 46$  and  $m\angle B = 82$ . Find  $m\angle ACD$ . Justify your results.



G. In  $\triangle ABC$ ,  $\overline{AB} \cong \overline{BC}$ . If  $m\angle Y = 112$ , what is the measure of  $\angle X$ ? Justify your results.



H. In  $\triangle SAT$ ,  $m\angle S = (2x - 10)$ ,  $m\angle A = (x + 15)$ , and  $m\angle T = (4x - 20)$ . Describe  $\triangle SAT$ .

I. The sides of  $\triangle PQT$  are 17.6, 11.7, and 9.6 meters. Find the perimeter of the triangle formed by connecting the midpoints of the sides of  $\triangle PQT$ . Justify your results.

*Vocabulary  
Concepts  
Skills*

- Rectangle
- Parallelogram
- Square
- Rhombus
- Kite
- Trapezoid
- Isosceles Trapezoid
- Diagonals
- Consecutive Angles
- Opposite Angles
- Opposite Sides
- Slope
- Parallel
- Perpendicular
- Congruent
- Similar
- Base
- Height
- Midsegment (median)

## 2.03 Apply properties, definitions, and theorems of plane figures to solve problems and write proofs.

### *b) Quadrilaterals*

A. The vertices of ABCD are A(-5, 1), B(3, 6), C(7, 0), and D(-1, -5). What is the best name for the figure? Justify your answer.

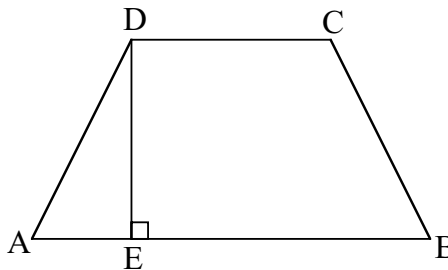
B. Find the length of the midsegment of the trapezoid with vertices (-3, -2), (-2, 1), (4, 5), and (1, -4). Is the trapezoid isosceles? Justify.

C. For parallelogram ABCD,  $m\angle A = (8x - 16)$  and the measure of the exterior angle at C is  $(5x + 18)$ . Find  $m\angle B$ ; justify.

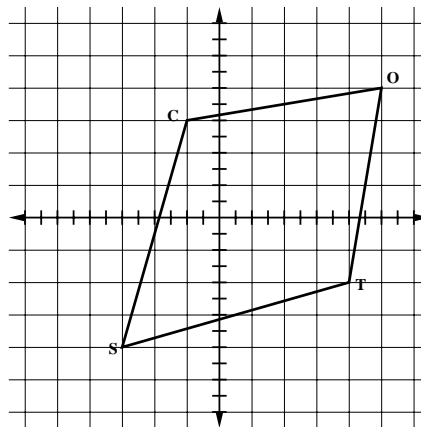
D. In parallelogram BCDF with A(3, -1) and D(-1, 5), find the coordinates of the point of intersection of the diagonals. Explain.

E. A parallelogram has vertices (-4, 5), (-1, -4), and (6, 4). What are the ordered pairs that can be the fourth vertex? Justify each.

F. In the diagram of isosceles trapezoid ABCD,  $m\angle A = 53$ ,  $DE = 6$ , and  $DC = 10$ . Find the perimeter of ABCD to the nearest tenth. Justify your results.



G. Which quadrilateral is TOCS? Justify.





Vocabulary  
Concepts  
Skills

Interior  
Angle

Exterior  
Angle

Regular

Irregular

Composite

Convex

Concave

Equilateral

Equiangular

Diagonal

Apothem

Inscribed

Circumscribed

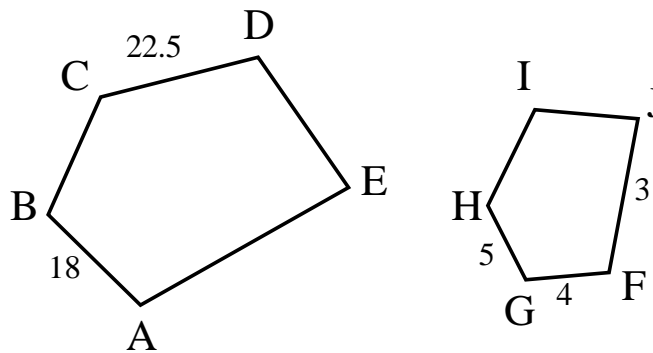
Polygons  
(3-, 4-, ... , 12-gons)

## 2.03 Apply properties, definitions, and theorems of plane figures to solve problems and write proofs.

### c) Other Polygons

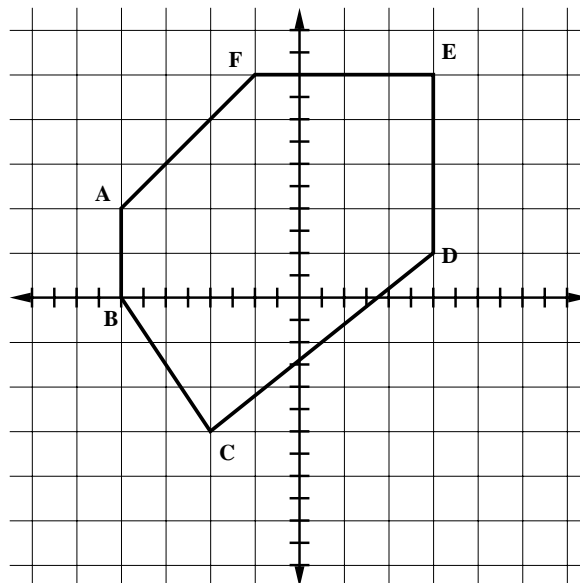
A. The measure of an exterior angle of a regular polygon is  $20^\circ$ . Find the number of sides of the polygon. Explain.

B.  $ABCDE \sim FGHIJ$ . Find the perimeter of each figure. Explain.



C. In convex pentagon  $ABCDE$ ,  $m\angle A = 6x$ ,  $m\angle B = (4x + 13)$ ,  $m\angle C = (x + 9)$ ,  $m\angle D = (2x - 8)$ , and  $m\angle E = (4x - 1)$ . What are the measures of all the angles? Justify your results.

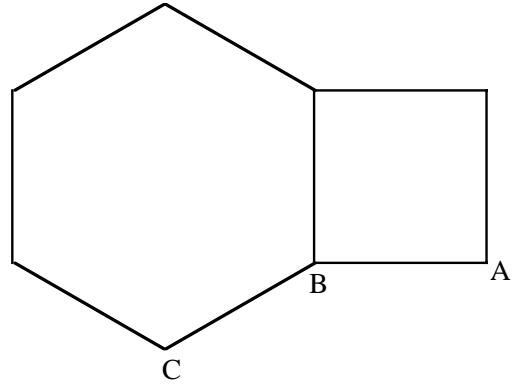
D. Find the measures of each of the exterior angles of  $ABCDEF$ . Justify your results.



**Vocabulary**  
**Concepts**  
**Skills**

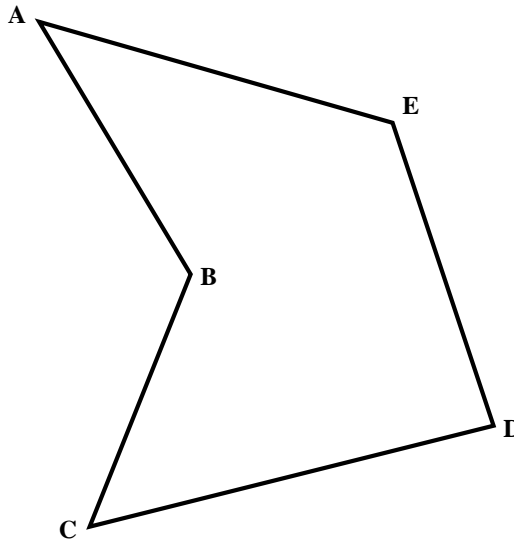
E. Find the apothem of a regular hexagon with sides of length 7.6 cm. Explain.

F. Point B is a mutual vertex of a regular hexagon, a square, and a third regular polygon (not completely shown). If two of the sides of this third polygon are  $\overline{AB}$  and  $\overline{BC}$ , what is this polygon? Justify.

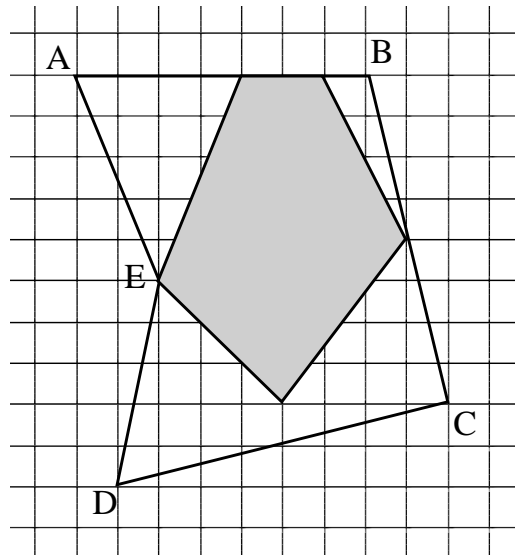


G. ABCDEF is a convex figure where  $\angle A \cong \angle D$ ,  $\angle B \cong \angle C \cong \angle E \cong \angle F$ , and the measure of the exterior angle at E is  $8^\circ$ . What are the measures of each of the interior angles?

H. Relocate vertex B so that ABCDE is convex and all sides remain the same length. Explain.



I. What is the probability that a point in the interior of ABCDE is in the unshaded area?



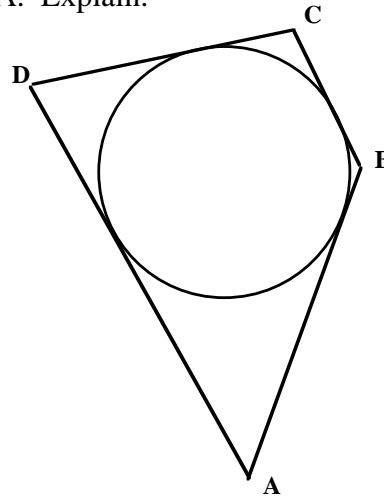
**Vocabulary  
Concepts  
Skills**

- Radius
- Diameter
- Circumference
- $\pi$
- Chord
- Tangent
- Secant
- Circumscribed
- Inscribed
- Concentric
- Major Arc
- Minor Arc
- Sector
- Semicircle
- Inscribed Angle
- Central Angle
- Internally Tangent
- Externally Tangent

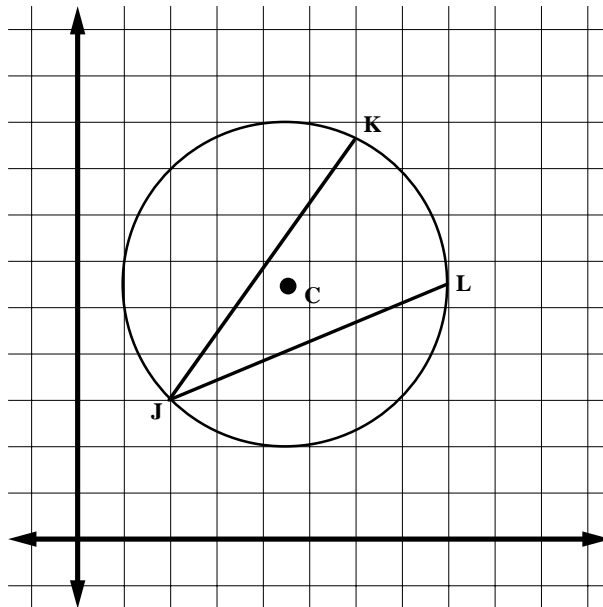
## 2.03 Apply properties, definitions, and theorems of plane figures to solve problems and write proofs.

### d) Circles

A.  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{DA}$  are tangent to the circle,  $AB = 14$ ,  $BC = 12$ , and  $CD = 16$ . Find  $DA$ . Explain.



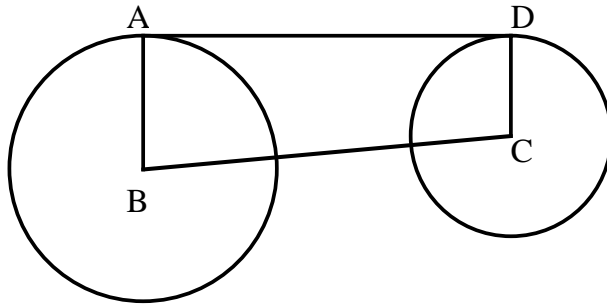
B. Given circle C, find the length of  $\widehat{KL}$  and  $m\angle KJL$ . Justify your results.



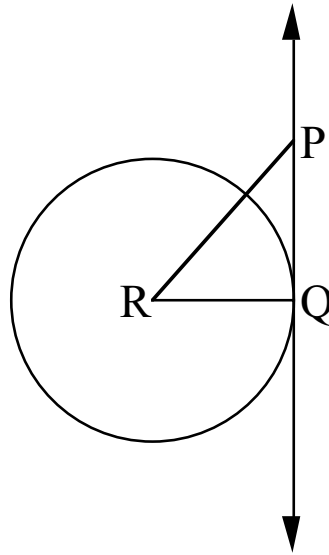
Updated 04/27/05

Vocabulary  
Concepts  
Skills

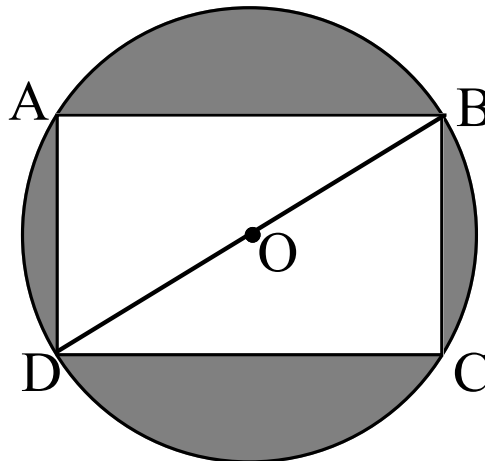
C. The radius of circle B is 38, the radius of circle C is 24 and  $AD = 91$ . Find  $BC$ . Explain.



D.  $\overleftrightarrow{PQ}$  is a tangent to circle R at point Q. The circle has a radius of 19. If  $m\angle R = 50$ , find  $RP$ . Justify your results.



E. In the diagram,  $ABCD$  is a rectangle inscribed in circle O. The ratio  $AB$  to  $BC$  is  $7:3$ . The area of the rectangle is  $165 \text{ cm}^2$ . Find the area of the shaded portion.

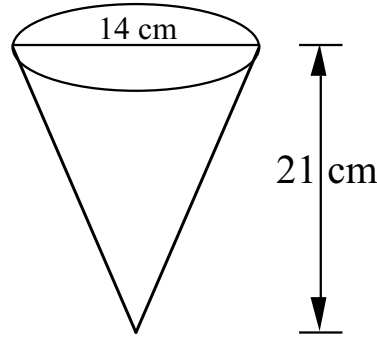


**Vocabulary**  
**Concepts**  
**Skills**

Edge  
 Face  
 Base  
 Vertices  
 Cube  
 Sphere  
 Cone  
 Cylinder  
 Prism  
 Composite  
 Truncated  
 Platonic Solids

## 2.04 Develop and apply properties of solids to solve problems.

A. If the diameter of the cone shown is increased by 2.5 cm, the volume of the new cone is what percent of the original?



B. A semi-regular polyhedron is a solid that has faces in the shape of more than one kind of regular polygon, each vertex is surrounded by the same kinds of polygons in the same order, and each edge is congruent.

Construct a truncated cube (a cube with its corners cut off) that is semiregular from a 4 by 4 by 4 cube.

What polyhedra is removed from each corner of the cube to form the truncated cube?

How long is each edge of the truncated cube?

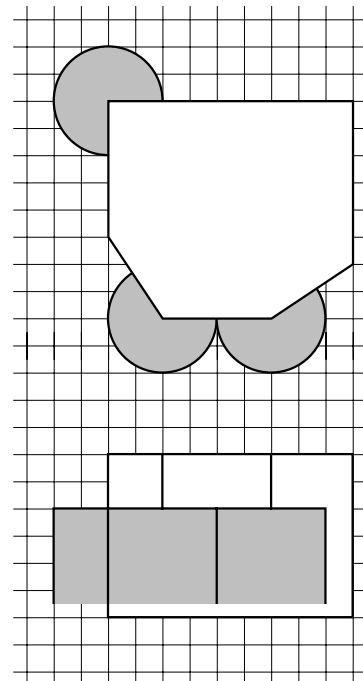
What polygons make the faces of the truncated cube?

What is the area of each face of the truncated cube?

What is the surface area of the truncated cube?

What is the volume of the truncated cube?

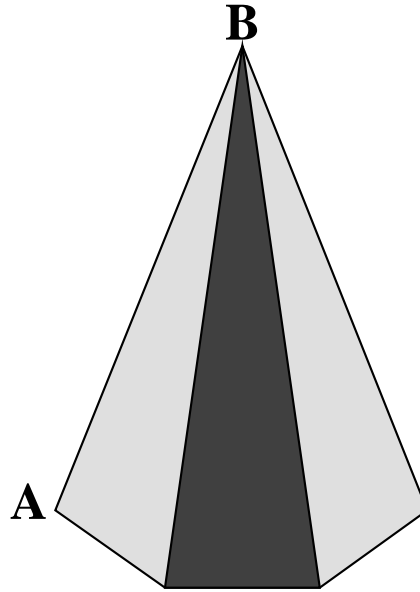
C. The top and side views of the new museum are shown. On the grid, 1 unit = 3 m. How much space will be heated or cooled during the year?



Updated 04/27/05

*Vocabulary*  
*Concepts*  
*Skills*

D. The base of the pyramid shown is a regular hexagon with length 13. If point O is the center of the base and OB is 15, what is the measure of  $\angle AOB$ ?



*Vocabulary  
Concepts  
Skills*

(Multiples of)  $90^\circ$   
Rotations

Center of Rotation

Center of Dilation

Mapping

Isometry

Clockwise

Counterclockwise

Pre-image

Image

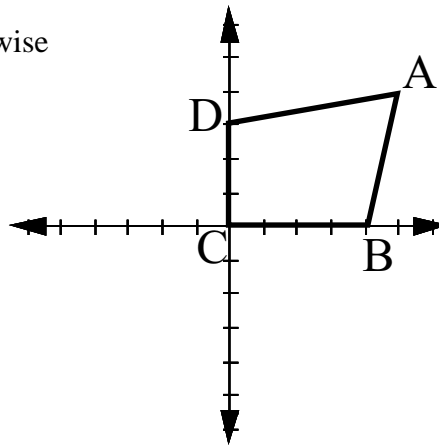
Composition

$$(x', y') = (ax + by + c, dx + ey + f)$$

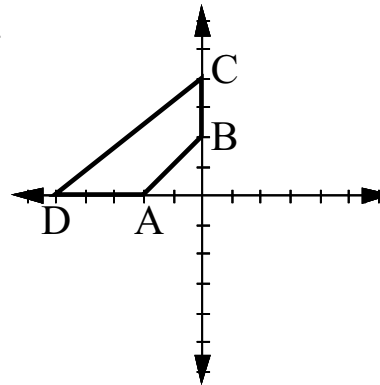
### 3.01 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

A. If  $\triangle A'C'B'$  was translated by  $(x', y') = (x + 2, y - 6)$  and the coordinates of  $\triangle A'C'B'$  are  $A'(-8, 9)$ ,  $C'(7, -3)$ , and  $D'(2, 6)$ , what were the coordinates of the pre-image?

B. ABCD is rotated  $270^\circ$  clockwise about the origin. Describe the transformation algebraically.



C. ABCD is reflected across the  $x$ -axis and translated five units to the left. Describe the transformation algebraically.



D.  $\triangle ABC$ , with vertices  $A(2, 8)$ ,  $B(5, 3)$ , and  $C(6, 8)$  is transformed according to  $(x', y') = (-2x + 3, y - 4)$ . Graph  $\triangle ABC$  and  $\triangle A'B'C'$ ; describe the transformation.

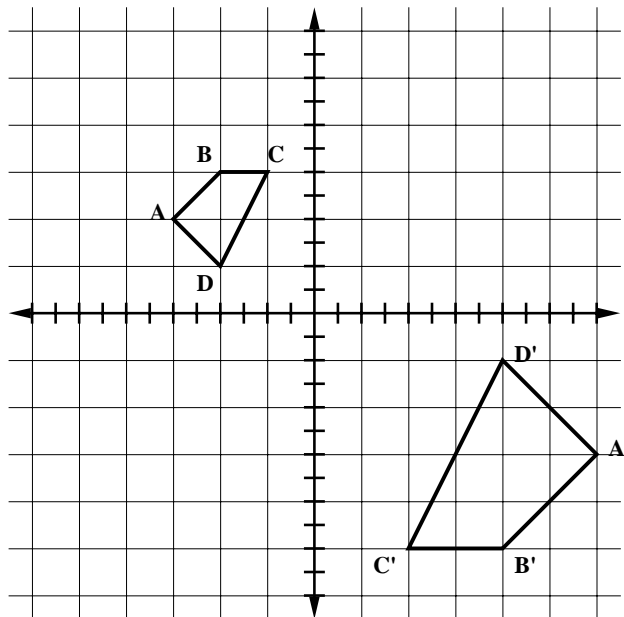
E. Dilate  $\triangle RST$ , with vertices  $R(2, 2)$ ,  $S(3, 6)$ , and  $T(8, 3)$ , by a factor of two and locate vertex  $S'$  at  $(3, 6)$ . Describe the transformation algebraically.

Updated 04/27/05

*Vocabulary*  
*Concepts*  
*Skills*

F.  $\triangle ABC$  has vertices  $A(9, 6)$ ,  $B(12, 3)$ , and  $C(6, -1)$ .  $\triangle PQR$  has vertices  $P(1, 6)$ ,  $Q(-2, 3)$ , and  $R(4, -1)$ . If  $\triangle PQR$  is the reflected image of  $\triangle ABC$ , what is the equation of the line of reflection? Write the algebraic expression that represents the transformation.

G. Algebraically describe the transformation of  $ABCD$  to  $A'B'C'D'$ .





*Vocabulary*  
*Concepts*  
*Skills*

Vertex  
Matrix

Standard Matrix  
Arrangement:  
alphanumeric left to  
right and top to  
bottom

Rows

Columns

Identity  
Matrix

Unit  
Matrix

## 3.02 Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

A.  $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$  is the vertex matrix representing  $\triangle ABC$ . Transformations of  $\triangle ABC$  are described in each expression. Evaluate each expression and describe  $\triangle A'B'C'$  with respect to  $\triangle ABC$ .

$$(1.) \ 2 \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$$

$$(2.) \ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 5 & 5 & 5 \end{bmatrix}$$

$$(3.) \ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & 1.5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(4.) \ \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$$

$$(5.) \ \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$$

$$(6.) \ \begin{bmatrix} 1.5 & 0 \\ 0 & 2.5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ 0 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

B. The vertices for  $\triangle MAT$  are  $M(-6, 8)$ ,  $A(3, 5)$ , and  $T(-1, -4)$ . Write a matrix expression that would rotate  $\triangle MAT$   $90^\circ$  counterclockwise.

C. The vertices for quadrilateral  $MNOP$  are  $M(-1, 3)$ ,  $N(-5, -1)$ ,  $O(-1, -2)$ , and  $P(3, 2)$ . Write a matrix expression that will shift  $MNOP$  six units left and four units down. What are the coordinates for  $O'$ ?  $MNOP$  is dilated by a factor of 1.4. Write the matrix that represents  $M'N'O'P'$ .

<p><b>Vocabulary</b></p> <p><b>Concepts</b></p> <p><b>Skills</b></p> <p>Mapping</p> <p>Isometry</p> <p>Pre-image</p> <p>Image</p> <p>Clockwise</p> <p>Counterclockwise</p> <p>(Multiples of) 90° Rotations</p> <p>Center of Rotation</p> <p>Translation</p> <p>Translation Vector</p> <p>Reflection</p> <p>Dilation</p> <p>Center of Dilation</p> <p>Composition</p>	<p>D. What is the matrix expression that will reflect <math>\triangle ABC</math>, <math>\begin{bmatrix} 2 &amp; -3 &amp; 1 \\ 4 &amp; 5 &amp; -5 \end{bmatrix}</math>, over the x-axis?</p> <p>What is the matrix expression that will reflect <math>\triangle ABC</math> over the y-axis and translate <math>\triangle A'B'C'</math> so that <math>B'</math> is at <math>(0, 0)</math>?</p> <p>Write the matrix expression that expands <math>\triangle ABC</math> horizontally by a factor of four and vertically by a factor of three.</p> <p>Write the matrix expression that dilates <math>\triangle ABC</math> by a factor of two and translates <math>\triangle A'B'C'</math> so that <math>A'</math> is at <math>(9, 3)</math>.</p> <p>E. Write the matrix expression that translates <math>\triangle ABC</math>, <math>\begin{bmatrix} 4 &amp; -7 &amp; 2 \\ 1 &amp; 1 &amp; -1 \end{bmatrix}</math>, so that <math>B'</math> is at <math>(-6, -2)</math>.</p>
--	---