The learner will demonstrate an understanding of linear relations and fundamental algebraic concepts.

5

Notes and textbook references

5.01 Identify, analyze, and create linear relations, sequences, and functions using symbols, graphs, tables, diagrams, and written descriptions.

A. Area and Perimeter Patterns (Blackline Masters V - 8 and V - 9) Students examine some geometric patterns to discover number patterns relating to perimeter and area. They then use these patterns to find formulas to predict perimeter and area for larger figures.

The patterns created from these figures are linear. It is fairly easy to use the repetitive pattern to find the answers required. Students may notice (or you may point this out) that patterns which increase by 2 every time n increases by 1 will have a formula of the form "$2n$ + ?". If the formula value increases by 4 each time n increases by 1, the formula will be of the form "$4n$ + ?". When you study slope, remind students of this example and explain how this ties in with slope.

Students should also be reminded of the geometric meaning of the variables. For instance in the first pattern, n represents the length of figure. Each figure has a top and bottom of equal length and ends of length 1. Pointing out this pattern, and then having students imagine what the 100th figure will look like, can help them find the formula. Show how this connects to the formula $2n + 2$ for perimeter.

Some students may be able to solve the problems by the iterative pattern only, others may know and use the rule about equal increment changes, others may see the pattern geometrically. Expose students to all three ways of viewing the problem and help them understand how they are connected.

Notes and textbook references

B. Creating Tables, Graphs and Equations from Authentic Situations Use Blackline Masters V - 31 through V - 34 in which students start with a written description and use it to create tables, graphs, and equations.

C. Toothpick Tasks (Blackline Masters V - 35 through V - 37) Have students build the shapes shown in Toothpick Task One using toothpicks or have them draw the shapes on grid paper. As students build the shapes have them fill in the table. Students should continue building the next shape in the sequence and listing the appropriate information in the table. As students complete several shapes they should begin to recognize how they are progressing from one term to the next and make predictions on future terms in the sequence without having to build the shape. Move students to writing a "word rule" for determining the perimeter and area of any shape in this sequence. Lead the students from the word rule to an algebraic expression. Students will come up with many different representations of the same expression when simplified. This lends itself to a great discussion about what the variable represents and how each student derived their rule. Follow the same procedures for Toothpick Tasks Two and Three. The expressions for finding the areas of the shapes in Tasks Two and Three are nonlinear. Discovering and writing these rules could be an extension or enrichment activity.

D. Patterns in Perimeter (Blackline Masters V - 42 and V - 43) Students will investigate, record and analyze perimeter data of various figures to create algebraic expressions that will allow them to determine the perimeter of any figure in a particular series. Allow students to work in pairs or small groups. You may have students build the patterns with toothpicks or pattern blocks. Have students share the expressions they create with the class. This will lead to a discussion of simplifying algebraic expressions as not all students will write their expressions in the same way, but they should all simplify to the same expression.

5.02 Translate among different representations of algebraic expressions, equations, and inequalities.

Notes and textbook references

A. Give pairs of students a set of cards found on Blackline Masters V - 10 and V - 11. Have students match equivalent expressions.

B. **Algebraic Expressions Square Puzzle** (Blackline Master V - 38) Students work to create a large square from the 16 small squares. Touching edges should contain equivalent expressions**.** Note: It would be best to cut out the small squares and place them in an envelope before giving the puzzle to the students, as the blackline gives the "answer".

C. **Heaps and Holes** (Blackline Masters V - 39 through V - 41) These sheets walk students through simplifying algebraic expressions through the use of diagrams. Some of the examples touch on the additive identity, multiplicative identity, and the distributive property.

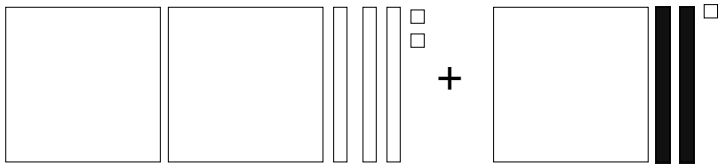
D. **Lining Up Dominoes** (Blackline Masters V - 44 and V - 45) Students will make a train of dominoes by simplifying algebraic expressions. It may be helpful to cut the dominoes apart and put them in envelopes ahead of time as the blackline is assembled as the "answer key". The blank domino sheet can be used for students to make their own dominoes. You may have them include other operations or concepts.

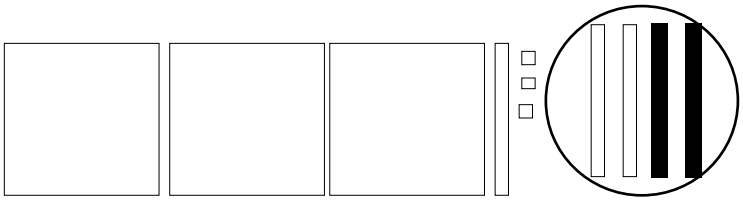
E. **How Do They Fit? # 1** (Blackline Masters V - 46 and V - 47) Students use their knowledge of computations with integers to solve equations. Allow students to work in pairs or small groups to assemble the 3 by 3 puzzle. Teachers may want to cut the puzzle pieces apart and put them in envelopes ahead of time as the blackline is assembled correctly. Use the blank blackline to create other puzzles that involve different skills or have students create their own puzzles.

Notes and textbook references

F. Use algebra tiles to demonstrate addition and subtraction of like terms in polynomials.

$(2x^2 + 3x + 2) + (x^2 - 2x + 1)$



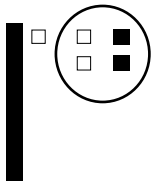


Because translating among various algebraic representations is abstract, students often have difficulty visualizing it. Using manipulatives such as Algeblocks™ or Algebra Lab Gear ™ can help students differentiate between variables such as x^2 and 2x. They can also be used to demonstrate algebraic properties such as the distributive property.

$3x^2 + x + 3$ is the result.

$(x^2 - 2x + 1) - (x^2 - x + 3)$

Take away **x^2**, **$-x$**, and add zero (+2 and -2).



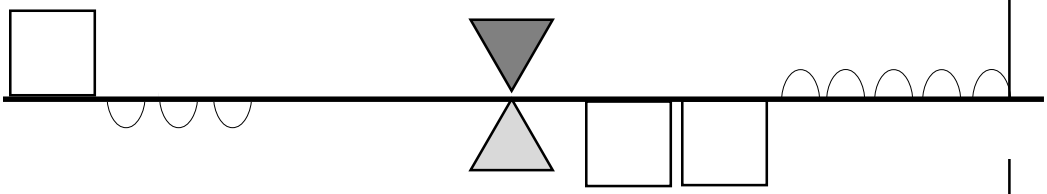
Now take away 3.

$(-x - 2)$ is the result.

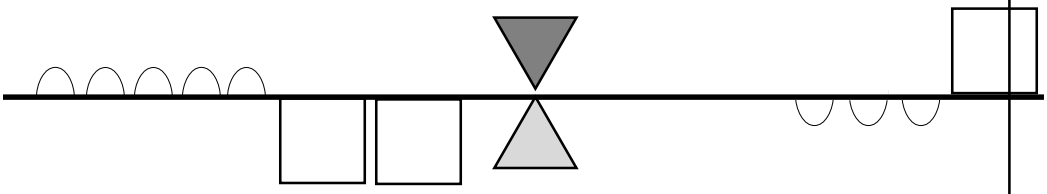
Notes and textbook references

G. Heaps and Holes - There are many manipulatives and drawing techniques to help students understand the properties of equality. Heaps and Holes diagrams can be used to illustrate that as well.
The notation that has been developed for Heaps and Holes can also be used to help solve equations. The ideas for solving equations would be similar to many other manipulative models you might have used.

Example: The equation, $x - 3 = -2x + 5$, can be illustrated as follows:

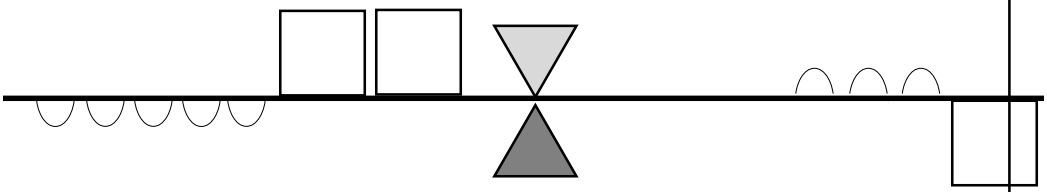


If we think of this as a seesaw in a room, we can transform it using a reflection about the fulcrum (vertical axis). This results in the following:



This could be an illustration of $-2x + 5 = x - 3$.
(Symmetric Property of Equality)

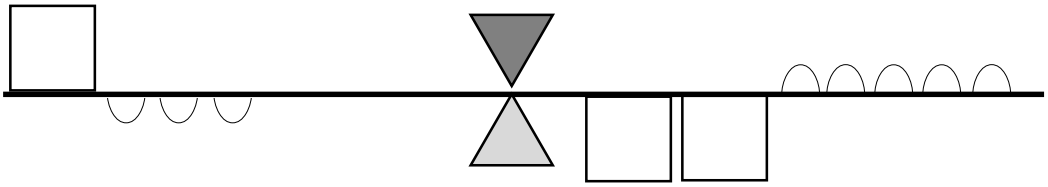
If we start with the result above and do a second reflection about the horizontal axis, the following results:



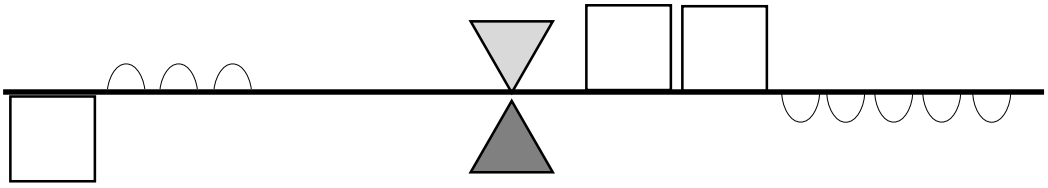
This could be an illustration of $2x - 5 = -x + 3$.
(Multiplying both sides by -1)

Notes and textbook references

If we start with the original, $x - 3 = -2x + 5$,



and do a reflection about the horizontal axis, the following results:

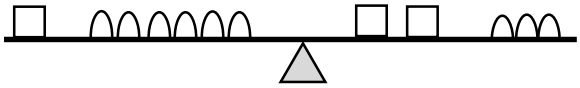


This is an illustration of $-x + 3 = 2x - 5$. (Multiplying both sides by -1)

Solving equations with Heaps and Holes diagrams:

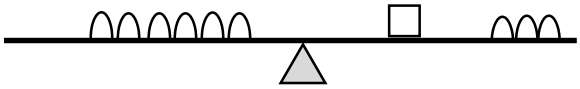
$x + 6 = 2x + 3$

$-x \qquad -x$

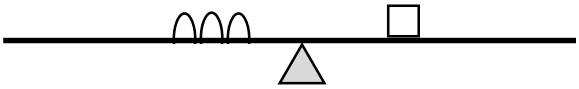


$6 = x + 3$

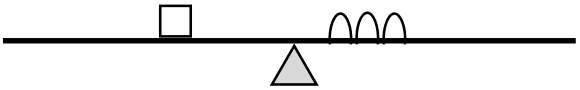
$-3 \qquad -3$



$3 = x$

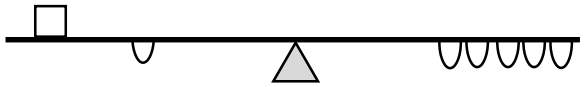


$x = 3$

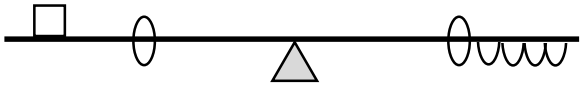


Notes and textbook references

$x - 1 = -5$
$+1 \quad +1$ Add 1

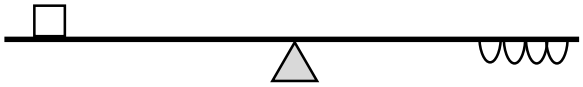


$x + 0 = -4$

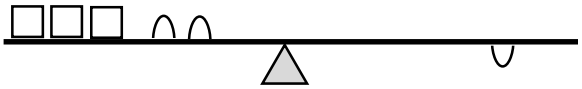


Additive Identity Property

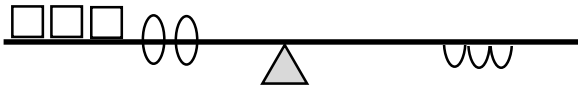
$x = -4$



$3x + 2 = -1$
$-2 \quad -2$ Subtract 2

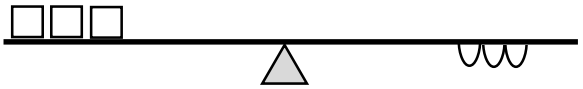


$3x + 0 = -3$



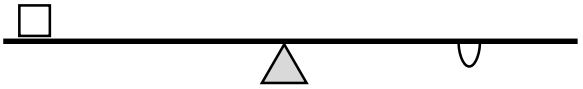
Addditive Identity Property

$3x = -3$



$\frac{3x}{3} = \frac{-3}{3}$ Divide into three equal groups

$x = -1$



H. How Do They Fit? # 2 (Blackline Masters V - 47 and V - 48)

Students use their knowledge of basic properties and computations with integers to solve multi-step equations. Allow students to work in pairs or small groups to assemble the 3 by 3 puzzle. Teachers may want to cut the puzzle pieces apart and put them in envelopes ahead of time as the blackline is assembled correctly. Use the blank blackline to create other puzzles that involve different skills or have students create their own puzzles.

I. Equation Relays (Blackline Masters V - 49 and V - 50)

- Divide the students into teams of three. Number each student in each team.
- Distribute the activity sheets to each team.
- Student #1 should write the first step for solving the first equation and then pass the sheet to student #2. Student #2 should write the next step for solving the equation. The sheet should continue to be passed until the equation is solved.
- For the next equation have student #2 start the process, and so on.
- Continue until all ten equations are solved.

Notes and textbook references

5.03 Use and evaluate algebraic expressions, linear equations or inequalities to solve problems.

A. **Solving Equations Square Puzzle** (Blackline Master V - 1) Students reassemble 16 small squares into a large square. When completed, touching edges will contain a two-step equation and its solution.

B. **Solving Inequalities Square Puzzle** (Blackline Master V - 2) Students reassemble nine small squares into a large square. When completed, touching edges will contain an inequality and its solution.

C. **One-Step Equations Triangle Puzzle** (Blackline Master V - 3) Students assemble a puzzle by matching sides of triangles. Where sides touch, an equation should match its solution. When the puzzle is correctly completed, the shape will be the figure shown in miniature on the page.

D. **Cooperative Problem-Solving Cards - Patterns** (Blackline Masters V - 4 through V - 7) Each problem is presented on a set of three or four cards. Students should work in groups with each student contributing the information on his/her card. Cooperative problem solving allows the students to witness the thought processes of their peers.

These problems contain patterns including integers, fractions, linear functions, ratio and proportion.

E. **Class President** -- Ask students to solve the problem below:

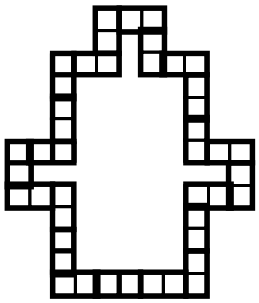
A teacher decides to pick a class president using the following technique. The students are all seated in a circle. The teacher then decides where to begin (call this chair 1) and continues numbering the students in a clockwise direction. The teacher first taps the student in chair 1 on the head and then continues around the circle tapping every other student on the head. As each student is tapped, he/she leaves the circle. The teacher continues tapping. If she comes to an empty chair, she ignores that and continues tapping every other student she encounters. The last student seated is chosen as president. Determine which seat will hold the president if you know the number of students in the class.

Encourage the students to start with small numbers in the class and gradually increase until they see a pattern. The students should experiment with up to ten students before they try to determine a pattern.

Notes and textbook references

F. **Equation Dominoes** (Blackline Masters V - 12 and V - 13)

Students should work in groups to match each word problem to the equation which may be used to solve it. On each match, students should match the edges of the dominoes with the stars. Upon completion, the dominos will form a closed loop as shown here.



G. **Over 40** (Blackline Master V - 14)

In this activity, students spin to collect enough symbols to create an inequality. Up to four students may play on one mat. On a student's turn, he/she spins the spinner to collect a symbol. He/she will write the symbols on the mat in dry erase or on a scrap paper he/she keeps visible to all players.

When a player has enough symbols to make a true inequality, he or she will display the inequality for the group to view. If the inequality is true, the player wins points. The point score is a total of two points for each letter used and one point for any other symbol used. He then erases all the symbols used in this inequality and keeps the unused ones if any. If the inequality is not true, the player loses his symbols and the score remains unchanged.
Players alternate turns until one player reaches a score of 40 or more.

You may laminate the mat or put it in a plastic sleeve and let the students write on it with a dry erase marker; you may prefer to let students write their symbols and scores on scrap paper. All students should be able to see the symbol collection of each player.

Notes and textbook references

H. Inequality Matching (Blackline Masters V - 15 through V - 21)

Materials: Deck of word problem cards and deck of inequality cards. Students should work in groups of three or four at the same table. Students work together to match four different problems to each of the inequalities.

Distribute the inequality cards among the students. There are 12 different inequalities, so groups of four work well. Shuffle the problem cards and place the pile face down in the middle of the group. On a student's turn, he takes the top problem card and reads it to the group. The group puts their heads together to determine which inequality card matches the problem. When finished, each inequality card should have four matches.

Students will probably have some disagreement over some problems. They may especially disagree on whether a ">" or "<" symbol should be used in some cases. Students can benefit from listening to the rationale of other students as they are making sense of the inequality problems. One key in the understanding is to identify the role of the variable ***B*** on each card. Once it is clear what ***B*** represents, the problem is easier to match.

Challenge the students to write a new problem for each of the twelve inequalities.

I. Solving Equations (Blackline Master V - 30)

Students should be familiar with writing equations from word problems and solving them. It is imperative that you have the student first write the equation and then solve the problem – showing all work. Students should be familiar with using variables other than *x* and *y*. Encourage them to use variables that represent the problem. For example, if they are talking about girls, have them use ***G***, if they are talking about rings have them use ***R***. The blackline gives a couple examples of problems your students could solve.

5.04 Develop fluency in the use of formulas to solve problems.

Notes and textbook references

A. Handcrafted Snowboards (Blackline Master V - 22)

Students apply cost and profit formulas to the snowboard construction business. Many situations can be modeled with a fixed cost term and a variable cost term. The snowboard formula is one such example. The questions in this activity require students to think about how changes in the situation will cause changes in the formula. They are also asked to use the linear formula to solve for both costs and production numbers. In other questions, they are asked to use the cost information to create a new formula for profit.

B. Music Lover's Special (Blackline Master V - 23)

Students use a spending formula to answer questions about a music lover on a shopping spree. This is a linear formula with several variables. The questions give students opportunities to solve problems involving maximum purchase possible from various situations.

Students are encouraged to use the formula to write their own challenging problems. Problem creation provides insight into and confidence in problem solving. When students have created their own problems, have students exchange these problems to see whether they have "stumped" others in the class.

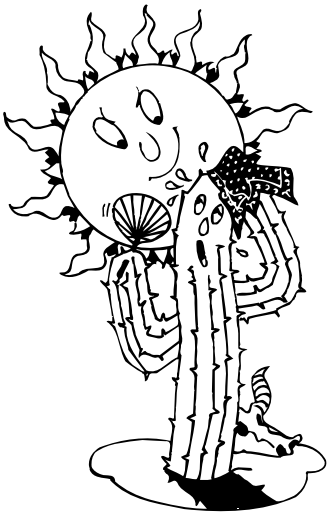
C. Fast Formula (Blackline Master V - 24)

Students use a given quadratic (2nd degree) formula to answer questions about Jeff on the waterslide. Students are asked to substitute directly into the formula to find heights at particular times. The students are then asked to find out how far he fell within various time intervals. They will be able to see that the formula has him falling faster and faster as he slides down the ramp. On a few questions the students are asked to estimate the time it would take to reach a particular height. This will involve calculating or estimating square roots.

D. Cook's Trick (Blackline Master V - 25)

This blackline demonstrates how the size of portions can be exaggerated by change of plate shape. The forumula for the area of an ellipse is used.

Notes and textbook references



E. **Prickly Gift** (Blackline Master V - 29)

Students use a rule about alien cactus reproduction to find a pattern and a formula for the cactus population. Students may find the solution to this puzzle in a variety of ways. Students often try to find a pattern by checking for consecutive differences. In this case, the differences are not constant, but they are powers of two. When this happens, the formula will involve powers of two. Students may find the answer by comparing the numbers in the 2nd column with the ones in the first. The numbers in the 2nd column are not powers of two, but they are always one less than a power of two.

Don't give too many hints. Let students find the solution and share their strategies with each other.

F. **Towers of Hanoi** (Blackline Masters V - 26 through V - 28) solves an ancient puzzle of time by working from a simpler problem to a more complex one. Students develop a forumla involving powers of two.