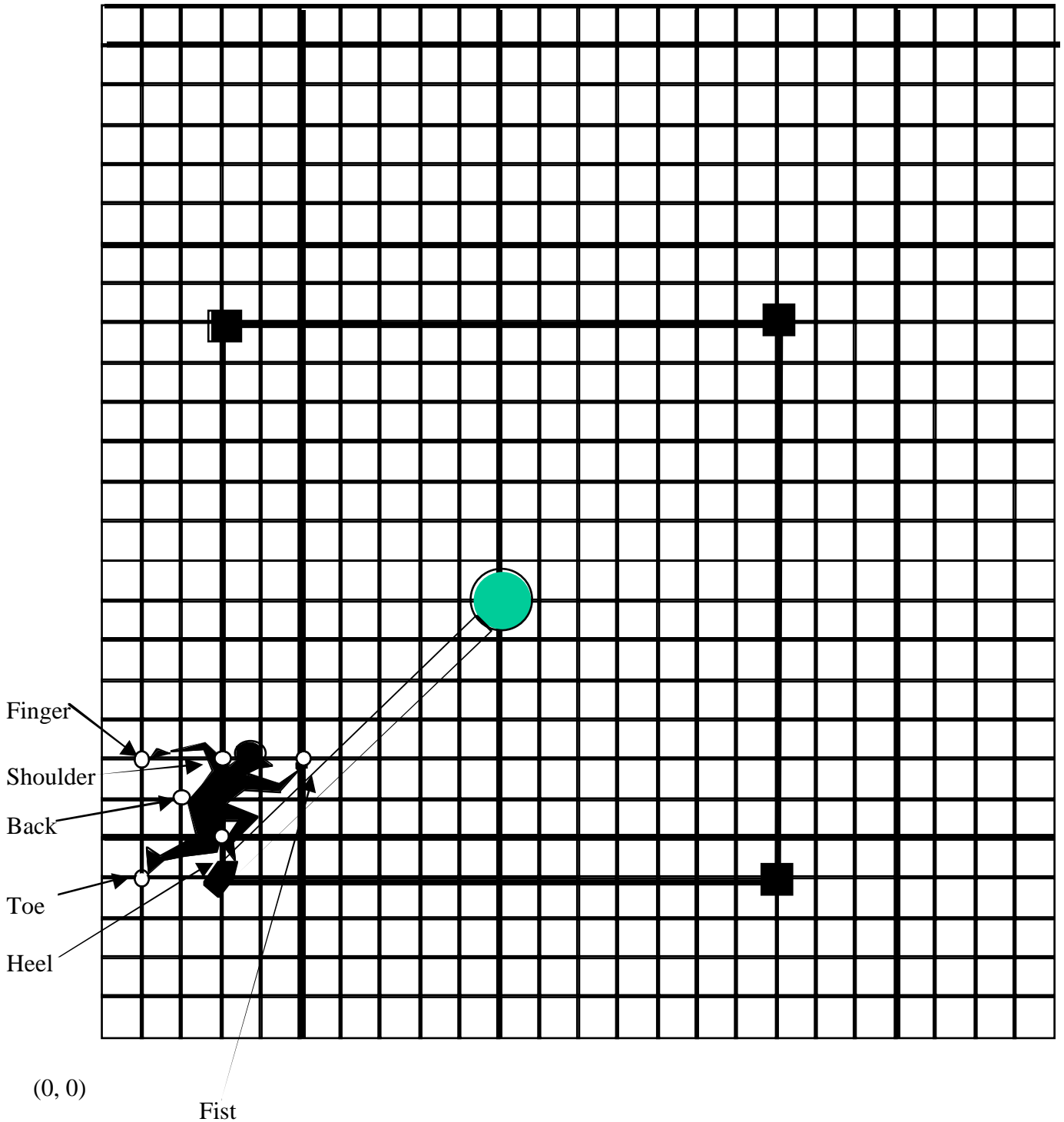
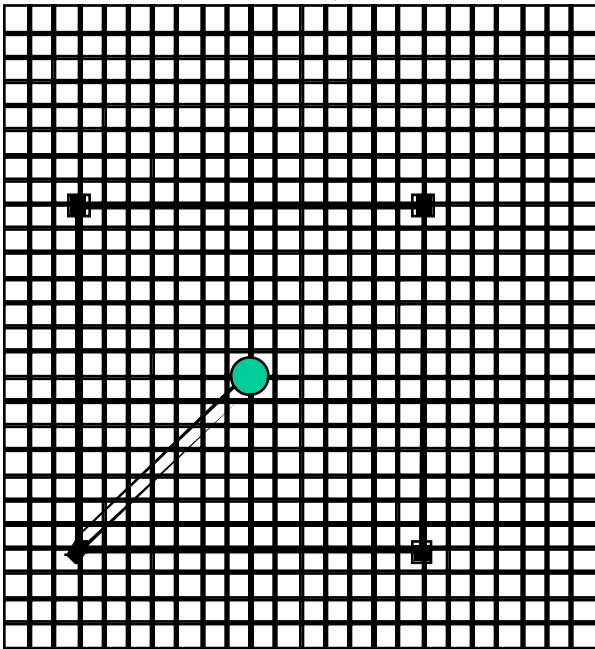


Slammin' Sammy

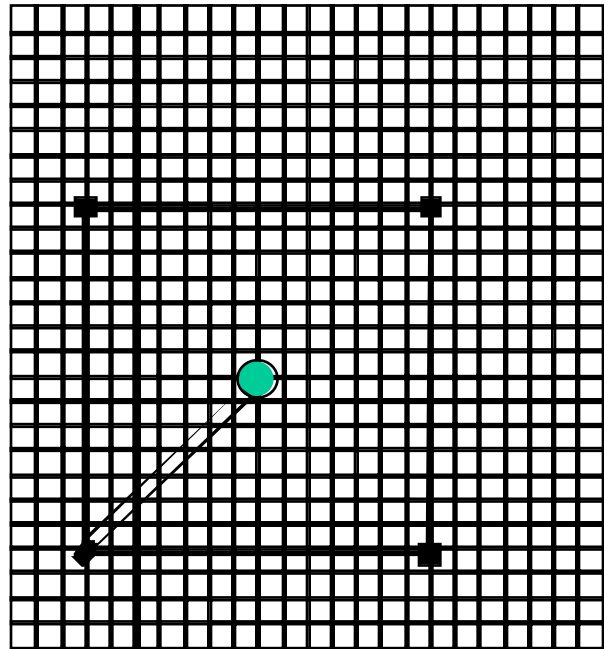


1. Give the coordinates of Sammy's six body parts:
 Finger (,) Shoulder (,) Back (,) Toe (,) Heel (,) Fist (,)

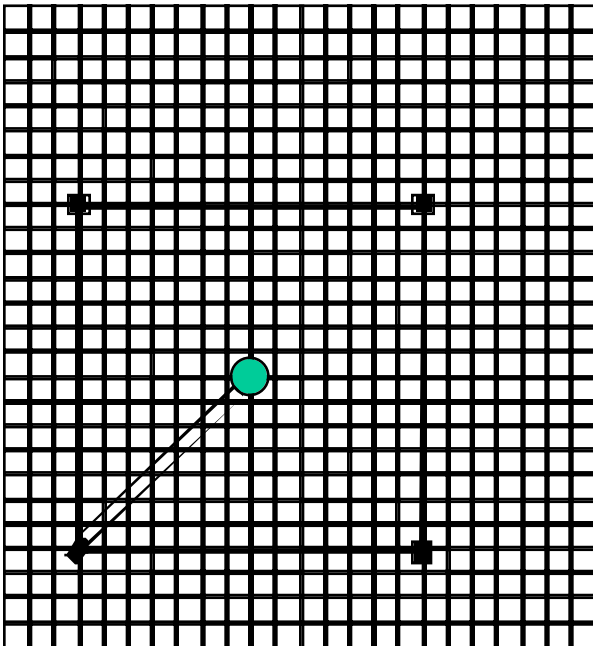
Slammin' Sammy



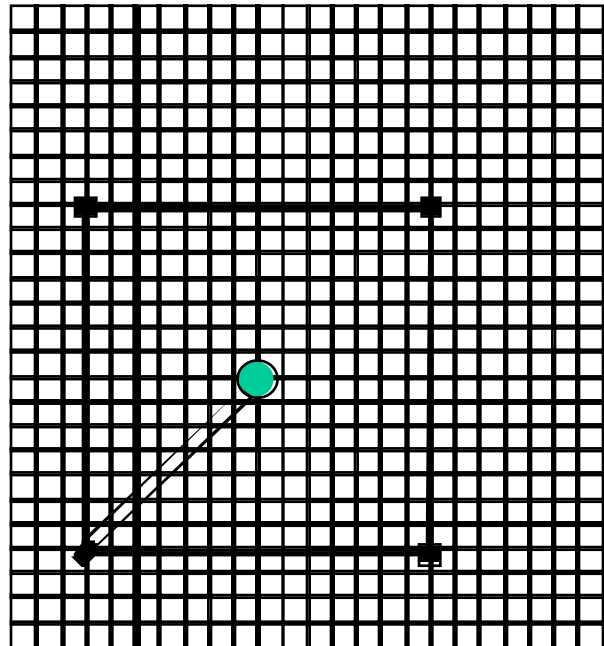
2. Draw Sammy at 1st base and give the coordinates of his five other body parts.
Toe (15, 4)



3. Draw Sammy at 3rd base and give the coordinates of his five other body parts.
Toe (1, 18)

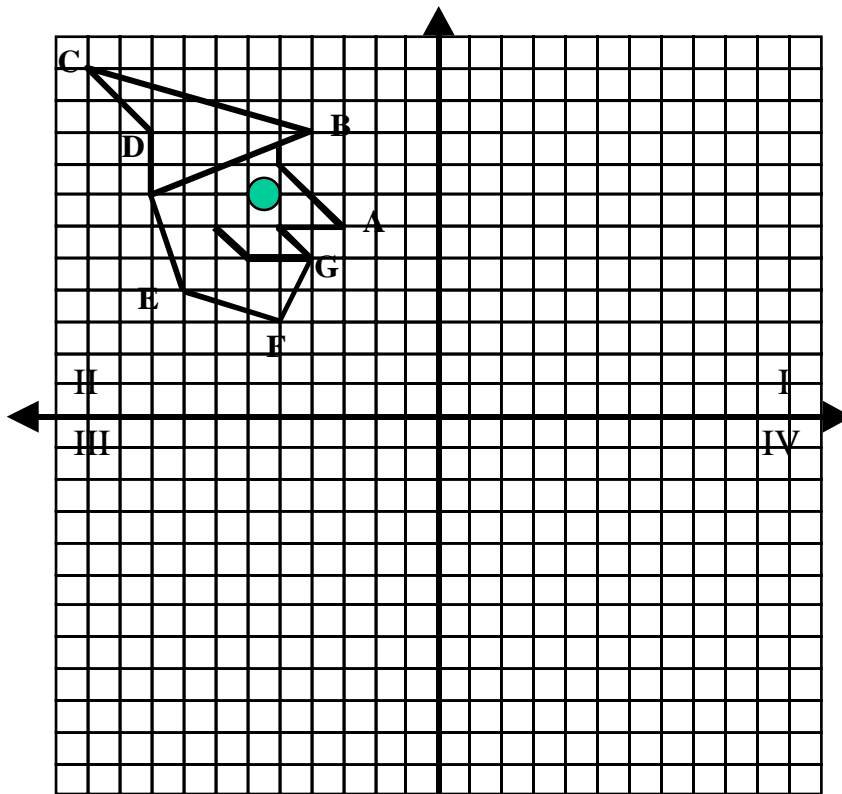


4. Draw Sammy at 2nd base and give the coordinates of his five other body parts.
Toe (17, 18)



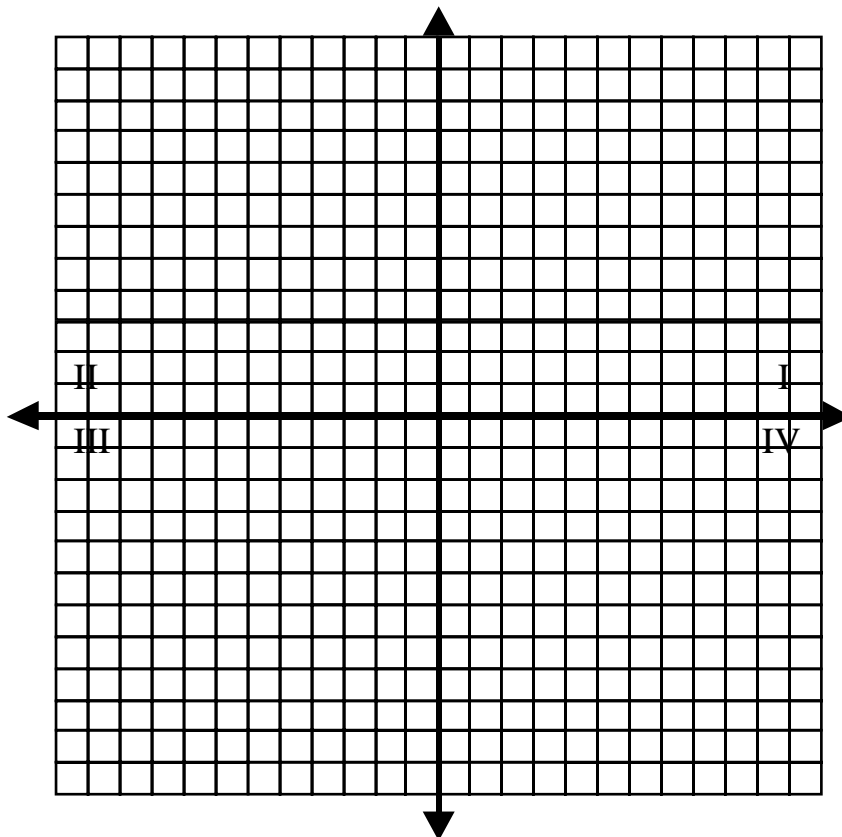
4. Draw Sammy at 2nd base but this time Reflect him to face 3rd base.
Toe (17, 18)

Draw it Again, Sam



Slide Sam into quadrant IV. Then flip Sam upside down into quadrant III. List the coordinates.

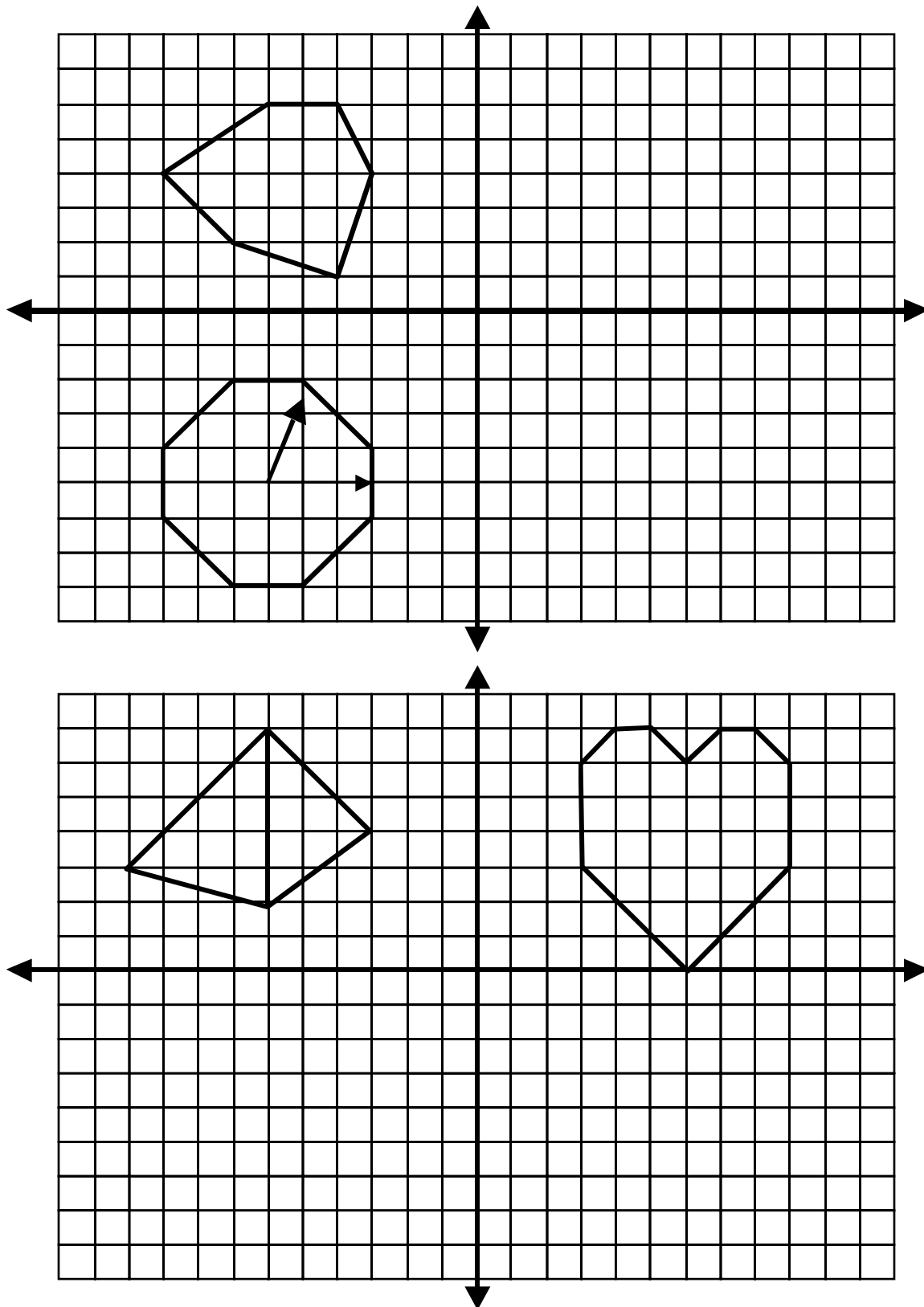
<u>II</u>	<u>IV</u>	<u>III</u>
A (-3,6)	(9, -6)	(-3, -6)
B _____	_____	_____
C _____	_____	_____
D _____	_____	_____
E _____	_____	_____
F _____	_____	_____
G _____	_____	_____



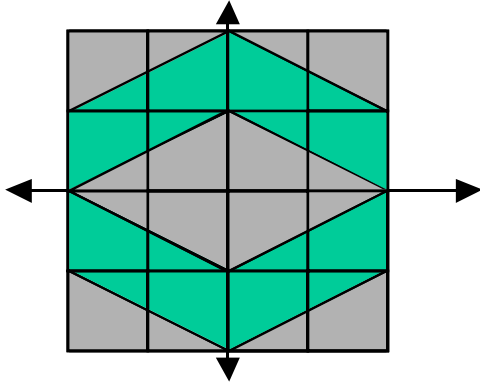
Now draw your own design in quadrant II. Slide it into quadrant IV. Then flip it upside down into quadrant III. List the coordinates.

<u>I</u>	<u>IV</u>	<u>III</u>
A _____	_____	_____
B _____	_____	_____
C _____	_____	_____
D _____	_____	_____
E _____	_____	_____
F _____	_____	_____
G _____	_____	_____

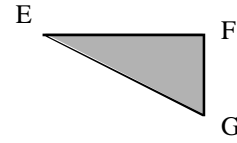
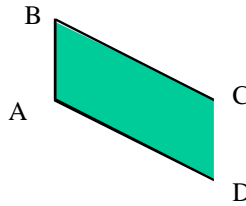
MIRA™ Activity



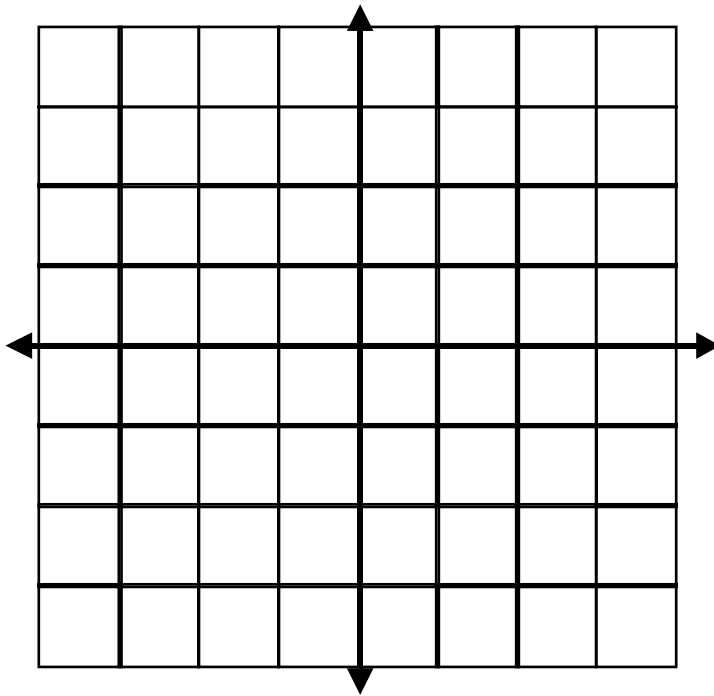
Quilter Challenge



Here is one way to create a design on a grid using the two shapes shown below.



Use the shapes to create your own design.
List the coordinates of each triangle and each parallelogram used.



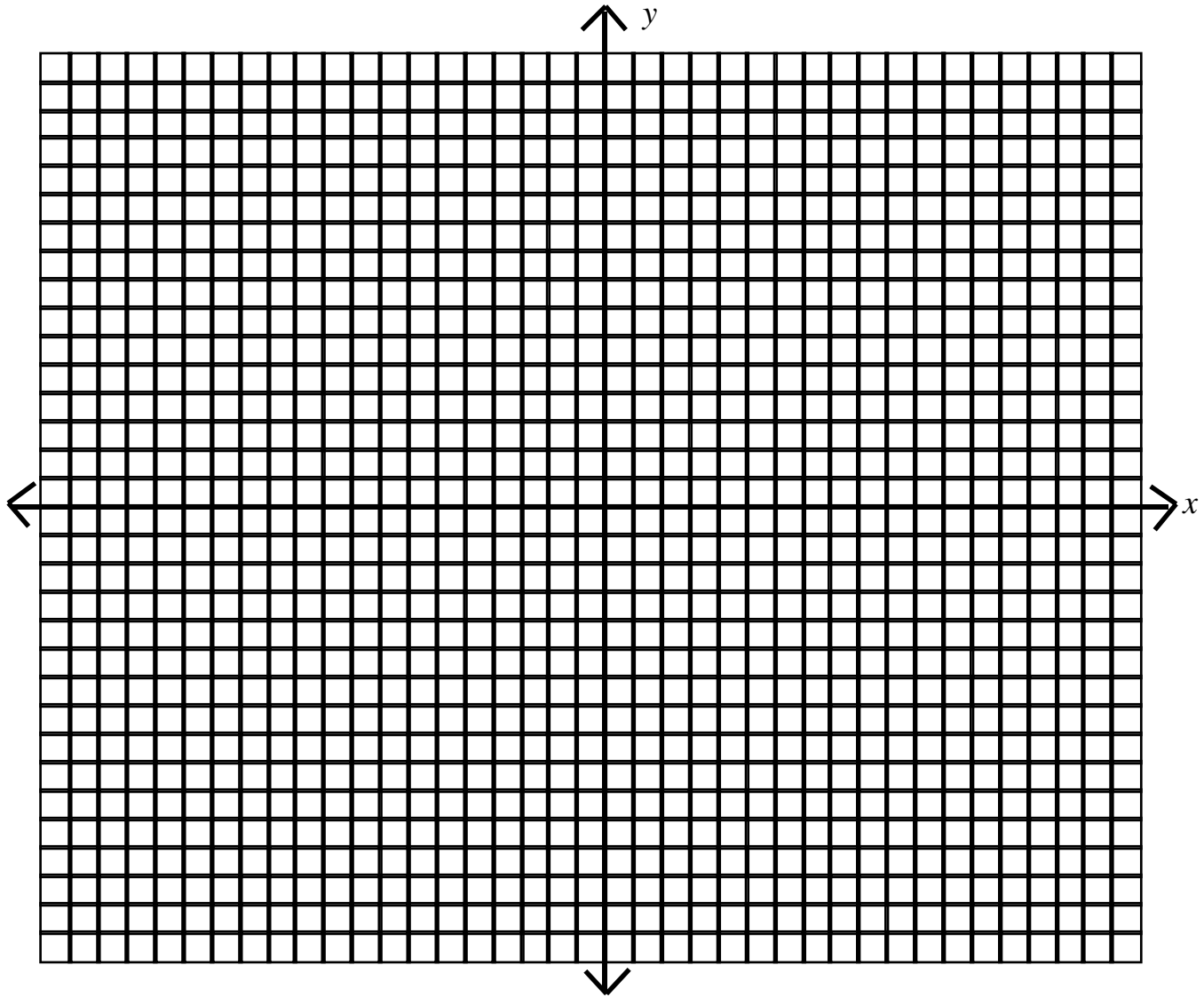
Parallelogram coordinates

A B C D

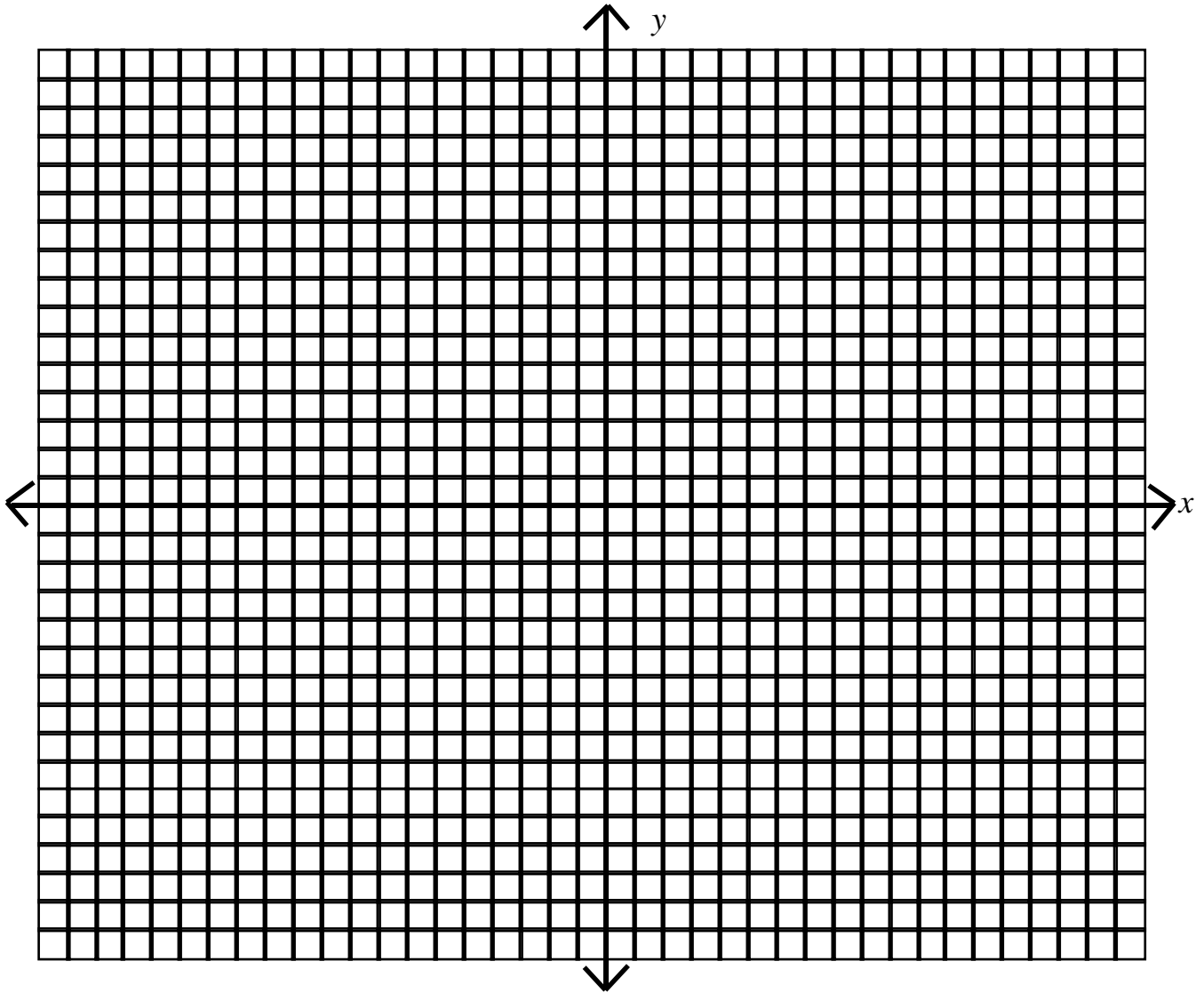
Triangle Coordinates

E F G

1. Connect these points in order to make a quadrilateral. $(-1,2)$, $(7, -3)$, $(15,2)$, $(7, 10)$
2. Draw the quadrilateral after it has been reflected over the y - axis. Draw this quadrilateral in red. List the new coordinates.
3. Reflect the original quadrilateral over the x -axis. Draw this quadrilateral in blue. List the new coordinates.

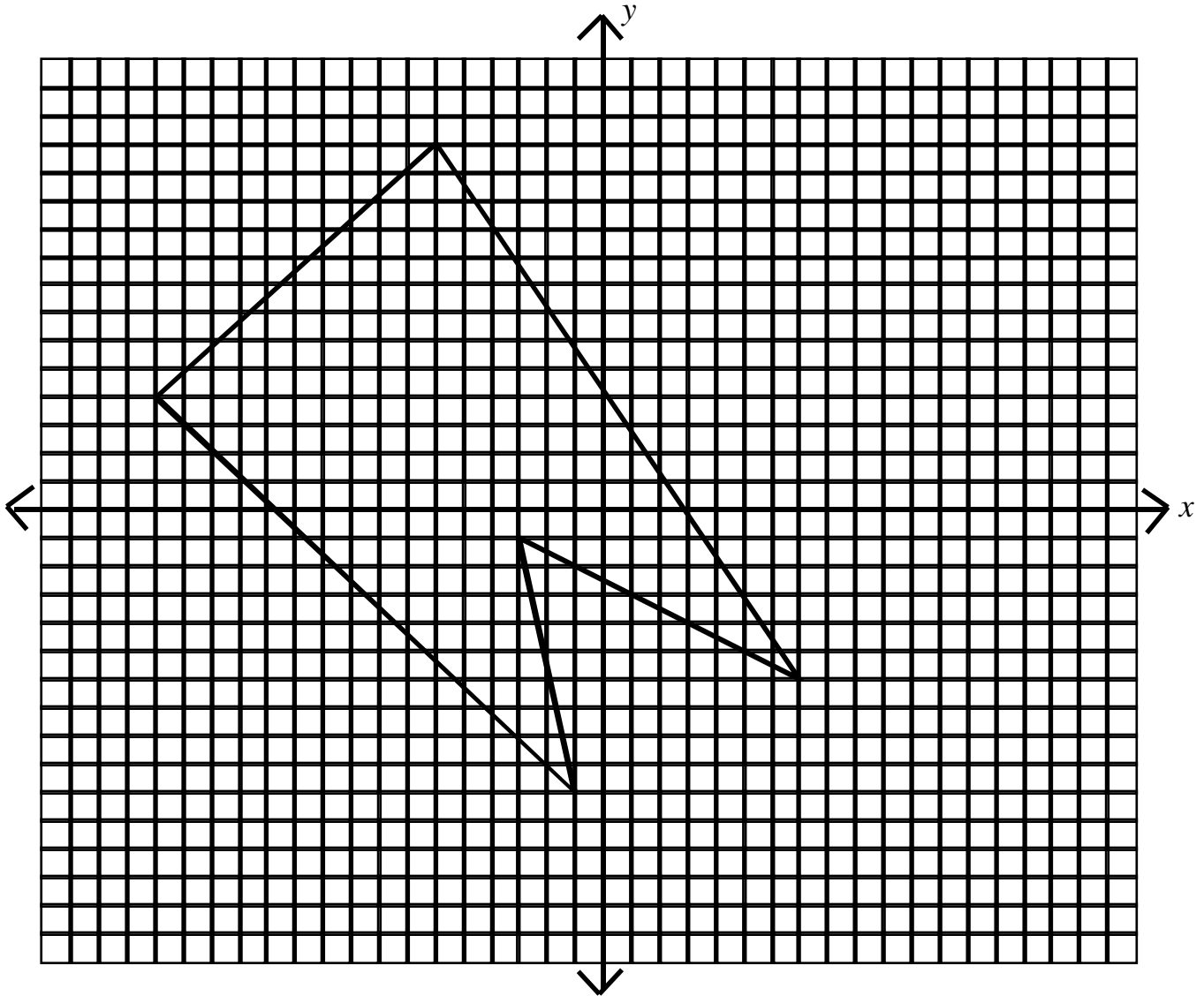


1. Connect these points in order to make a quadrilateral:
(-12,-2), (-12,10), (-6,10), (-6, 5), (-12,5)
2. Translate the figure according to the rule: $(x, y) \rightarrow (x + 8, y - 10)$



1. Draw the pentagon below after it has been reflected over the y - axis. Draw this figure in red.

2. Draw the pentagon below after it has been reflected over the x -axis. Draw this figure in blue.



Transformations in the Coordinate Plane

Joseph Georgeson

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In the context of transformations, students will explore the effect addition and multiplication have on shapes drawn in the coordinate plane. Why is it, for example, that when something is added to the coordinates of a shape, it is not changed in size, only in position? Or how does multiplication of the coordinates of a shape affect the shape if the number is positive, negative, bigger than 1, less than 1 but bigger than 0, or equal to 1, or the result of adding numbers that are less than 0 to the coordinates?

The activities should be set up so that students are given the instructions for making a transformation rule. Their job is to work in cooperative groups and attempt to make conjectures about their rules, test the validity of their conjectures, and demonstrate to other groups what they have found. It is hoped that through the activities in this unit, students will come to a better understanding of arithmetic as it applies to this geometric model, and will have a better appreciation for the transformations that they see in the world around them.

Prerequisite :

- coordinate geometry (knowing how to locate points when the coordinates are given)
- area and perimeter on grid paper (informal knowledge is all that is required)
- other skills that students either have learned and forgotten, or never have learned at all

(Note: This is always the case in any class, so it is assumed that the teacher will provide appropriate instruction when necessary. For example, if students have never used variables, they will need some instruction. Some students, however, may have had exposure to variables, but in a different context. The teacher must determine what enabling instruction needs to be given.)

A discussion of the word "transformation" would be a good place to begin this unit. A general definition of transformation might be "change." Examples to be considered are slides, turns, flips, expansions, contractions, or combinations of these. The vocabulary that is used should be whatever students can understand at an intuitive level.

Students should be able to give many examples of transformations that they see around them. Have them give examples while a list is kept on the overhead. Keep that list during the time the unit is being done for reference. A few examples that might be mentioned (you should spend time informally discussing these examples and solicit from students many more examples):

- bricks on a wall, ceiling tiles, desks in a classroom, and blocks on a sidewalk can all be modeled by a translation or slide;

Transformations in the Coordinate Plane

- propellers, certain letters of the alphabet, and Ferris wheels are examples of turns or rotations;
- people growing, the image of a shape on an overhead projector, or the image of a slide in a slide projector are all models of an expansion;
- melting ice, maps of the world, and model airplanes are models of contractions; and mirrors, cars, people, and many geometric shapes model reflections or flips.

This list is not complete. Students should add to this list from their own experiences. Once they start, they will find many examples. This could be an ongoing activity from the beginning of the unit until the end of the year.

Activity 1:

On the first day bring magazines to class and have students identify pictures that represent each of the transformations. Make a bulletin board on which a collage of examples of the various transformations are displayed. Students could work in groups and put their pictures on the board in an organized fashion. One group could be responsible for “putting up” rotations or turns, while another group might be responsible for translations. Every group will find examples and funnel them to the appropriate group to decide how to display all of them.

End the class with discussion of their examples. Many questions could be asked, such as how they identified the transformation. What characteristics make a translation? How can you describe the rotation you found? What is special about a flip or reflection that tells you that it is one?

Activity 2:

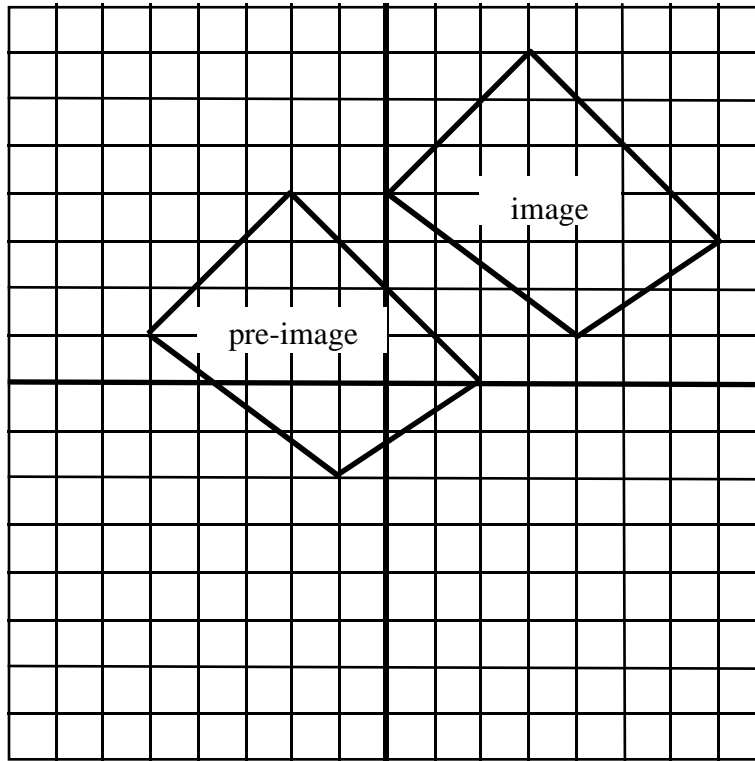
The class will be exploring the transformations that they identified and displayed in Activity 1. However, they will be doing the exploration on graph paper using mathematical notation to describe each transformation. The same mathematics that works on graph paper is applicable to the real world models of transformations they have found. It is important to relate this mathematical model to the real world examples as often as possible. The following example introduces the notation that students will use during this unit. It gives students a working vocabulary to apply to each task so that they will understand what they are looking for when they work independently.

Transformations in the Coordinate Plane

Vocabulary: pre-image — the original shape
 image — the transformed shape

Example 1: Demonstrate the following transformation:

$(x,y) \longrightarrow (x + 5, y + 3)$ means add 5 to the x-coordinate and 3 to the y-coordinate for each of the vertices in the pre-image to build the image.



For example, the point $(-1, -2)$ is transformed into the point $(4, 1)$ using the rule, “add 5 to the first coordinate, and add 3 to the second coordinate.” The other vertices of the pre-image are transformed in the same way.

Note to teachers: Shapes can be restricted any way the teacher feels is necessary. For example, restricting shapes to the first quadrant might be necessary for some classes, although, in some cases it might be surprising to see what students can do and understand if the context is meaningful. The context of these transformations could serve as a good model to introduce in a meaningful way the idea of adding positive and negative numbers.

Transformations in the Coordinate Plane

Questions:

- In what ways are the image and pre-image alike?
- How are they different?
- Did the same thing happen to every point, or just the corners?
- Are the shapes similar? Congruent?
- Are the lengths similar? Equal?
- What do the numbers 5 and 3 have to do with anything?
- What if 5 were subtracted instead of added?
- Why didn't the shape get bigger -- we added something to the coordinates?
- If corresponding vertices are connected, what shape results?
- What questions can you (the student) ask?
- Are there other things you noticed?
- Are the shapes oriented the same way? (Is the top still the top?)
- What if this transformation were done over and over again?
- Many more are possible. Students could surely come up with a few.

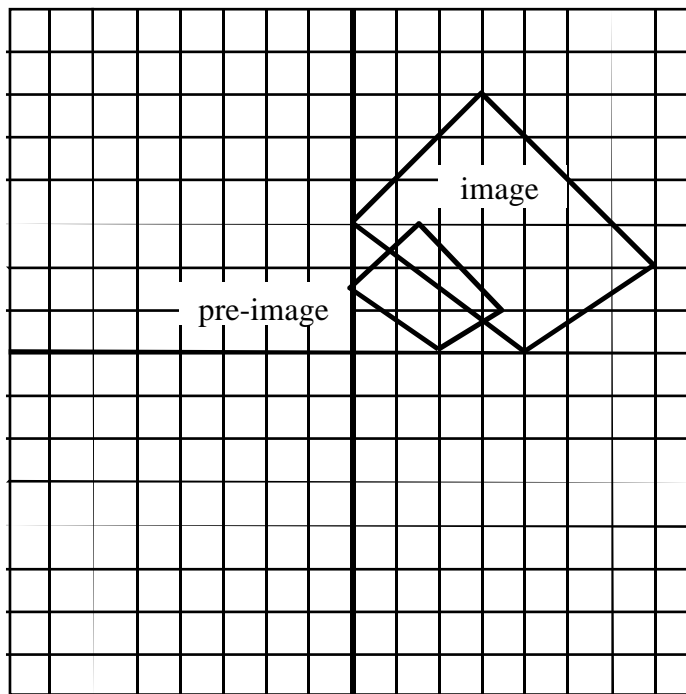
Next, ask students to predict what will happen in the following transformation:

$$[(x,y) \longrightarrow (x+4, y+4)]$$

- Why do you think that? How is this like the first one that was demonstrated?
- How could you verify your prediction?

Example 2: Demonstrate the following transformation:

$$(x,y) \longrightarrow (2x,2y)$$



Transformations in the Coordinate Plane

Notes:

The image of $(2,0)$ is $(4,0)$, and so on. The coordinates in the image are found by following the rule, "double each coordinate." It is important to both write the rule using algebra (symbols) and say the rule using everyday language. This helps students see that one use of a "variable" is to describe in a convenient way a pattern or rule that has some use.

Other shapes could be graphed. It is important to pick shapes that fit on the graph paper you are using. Students could make up their own. Be careful (or aware) of shapes like squares or rectangles with lines of symmetry that make it difficult to see certain transformations.

Questions:

Is this an expansion or a contraction (did the image get bigger or smaller)?

What happened to the shape?

How are the image and pre-image related (sides, angles, orientation)?

If corresponding vertices are connected, what shape results?

How is the area related? (This might be where a "guess" is acceptable, or simply the idea that the shape got bigger or smaller if the transformation is a contraction.)

What is the ratio of the length of the pre-image to the length of the image?

How do the areas compare when expressed as a ratio?

Activity 3:

Give students adequate time, plenty of graph paper, and some rules to investigate. After doing the few you recommended, they should make up some rules on their own. Their job as they do this activity is to discover relationships between what is done to the coordinates and its effect on the image. The features they should look for are size (area and perimeter), orientation (top, bottom, left, and right), shape (similar, congruent, stretched, distorted), and others that students feel are important to name in describing what happened to the shape after it was transformed. Which of these features remained the same, and which were changed? How were they changed? The vocabulary of the students and the way in which they described the transformation should be accepted. Try to make them feel that the discoveries they are making are new and different and that you are surprised.

Transformations in the Coordinate Plane

Suggested Transformations:

Students should investigate several classes of transformations:

- a. Transformations that lead to expansions will be of the form:
 $(x,y) \longrightarrow (ax,ay)$, where $a > 1$.
- b. Transformations that lead to contractions will be of the form:
 $(x,y) \longrightarrow (ax,ay)$, where $0 < a < 1$.
- c. Transformations that lead to reflections will be of the form:
 $(x,y) \longrightarrow (ax,ay)$, where $a < 0$. The size may change also.
- d. Transformations that lead to translations or slides will be of the form:
 $(x,y) \longrightarrow (x + a,y + b)$, where a and b are real numbers. Integers would be the easiest to consider.
- e. Transformations of the form $(x,y) \longrightarrow (y,x)$ will lead to reflections, but the line of reflection will be the line $y = x$.
- f. Other transformations that could be explored:
 $(x,y) \longrightarrow (0,y)$
 $(x,y) \longrightarrow (x,0)$
 $(x,y) \longrightarrow (x,y)$
 $(x,y) \longrightarrow (2,4)$

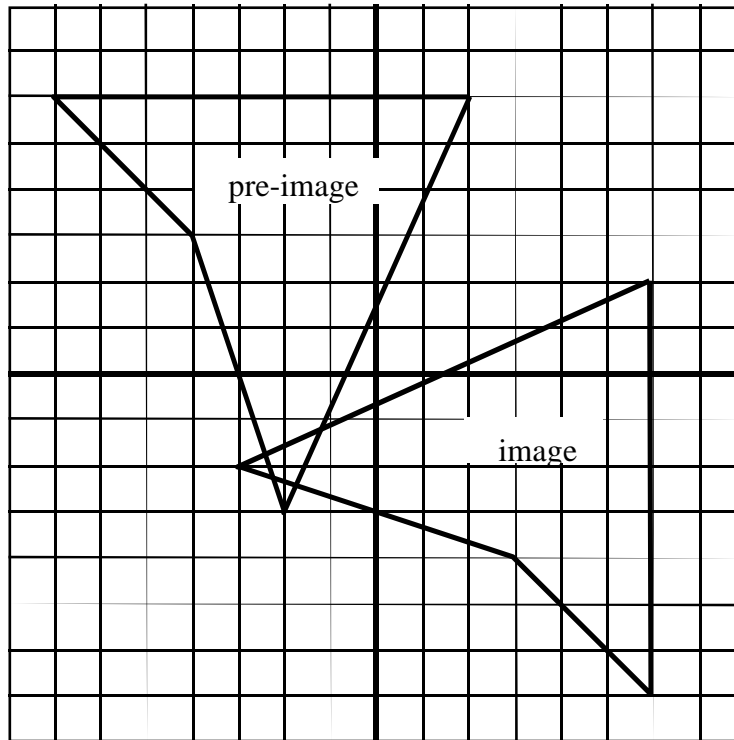
Summary Questions:

How does the image of a shape that is drawn change when the coordinates are changed according to some rule? What features of the shape are changed, and what features remain fixed? How could you describe the changes? How are the changes related to what was done to the coordinates? These are some questions that this unit attempts to answer. It is not important for all students to find all of the answers. It is, however, important for all students to explore these patterns and work with other students to better understand these ideas.

Transformations in the Coordinate Plane

Some examples of what students should be able to do.

$$(x,y) \longrightarrow (y,x)$$



$$(x,y) \longrightarrow (-x,-y)$$

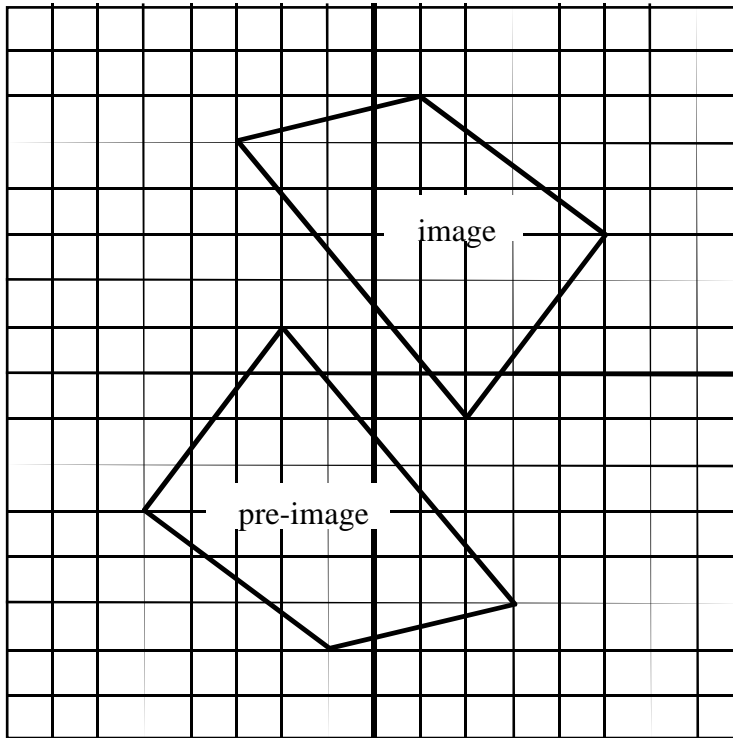


Table Top Transformations

Materials:

- A large piece of 1" grid graph paper from a roll or 4 sheets of 1" grid graph paper taped together to form on large sheet
- Markers
- Rulers
- Scissors
- Index cards
- Charts TTT-1 and TTT-2
- Graph grids to record transformations

- 1) Write in your math notebook anything you know about slides, flips, and turns.
- 2) Compare notes with a partner and make additions.
- 3) Investigate and discuss correct vocabulary (translations, reflections, and rotations).
- 4) Use your large sheet of graph paper to make a coordinate plane. Show x- and y- axes. Label grid from -12 to 12.
- 5) Cut geometric shapes described in BlacklineMaster III - 17 from 3" x 5" index cards.
- 6) Place the rectangle in the first quadrant with vertex A at (2,3) and B at (2, 6).
- 7) Record the other coordinates in TTT-1 for the original position.
- 8) Complete the chart for the rectangle's translation 3 right and 1 down, rotation 90 degrees clockwise, and reflection across the y-axis.
- 9) Remember to return to your original position after completing each transformation.
- 10) Repeat steps 7-10 for each polygon.
- 11) On separate graph paper, graph the original position, rotation, translation and reflection for each polygon.

Extensions:

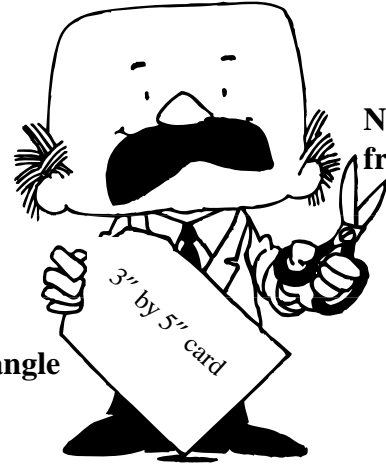
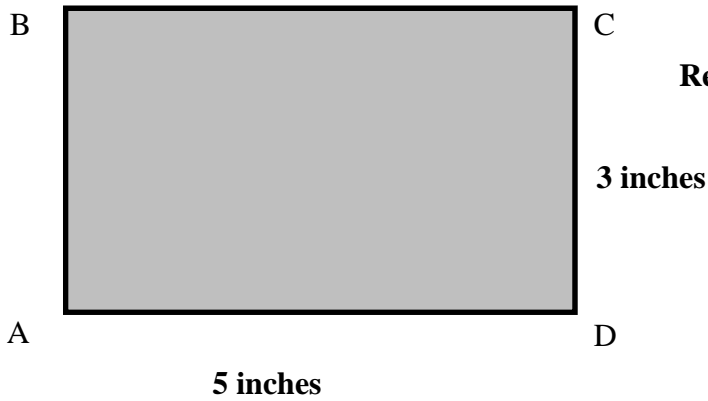
- Complete chart TTT-2
- Use different starting points, polygons, points of translation
- Rotate different degrees and directions
- Reflect across a line other than an axis, or rotate around a point other than the origin.
- Try a larger-than-life-size coordinate plane on the floor of your classroom. Use students to plot points.

Concepts to Review:

- Placement and labeling of quadrants
- Geometric shapes, names and properties
- Writing ordered pairs
- Location of x- and y-axes
- Counterclockwise and clockwise

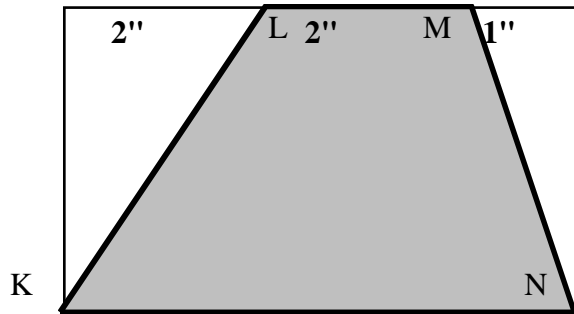
Table Top Transformations

Shapes to use:

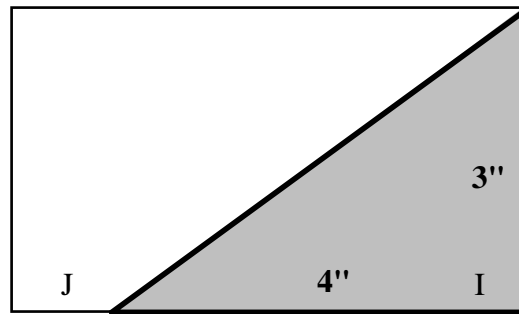


Note: cut these shapes from 3" by 5" cards

Trapezoid



Right Triangle



Isosceles Triangle

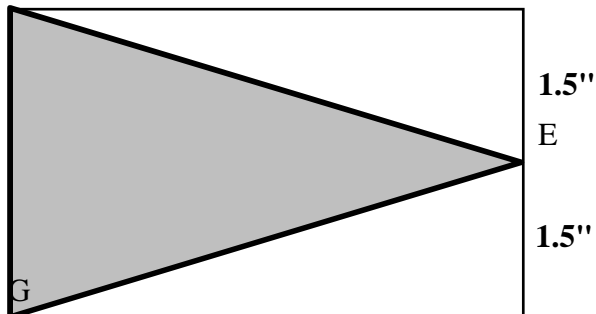


Table Top Transformations

Chart (TTT - 1)

Give coordinates of original position, then the coordinates of the transformed shape. The rotation is 90° clockwise about the origin. The reflection is about the y-axis.

Shape	Original Position	Translate 3 right, 1 down	Rotate 90° Clockwise	Reflect across y-axis
Rectangle	A(2, 3) B(2, 6) C(,) D(,)	A(,) B(2, -4) C(,) D(,)	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)
Right Triangle	H(0, 3) I (0, 0) J (,)	H(,) I (2, -4) J (,)	H(,) I (,) J (,)	H(,) I (,) J (,)
Isosceles Triangle	E(-2.5, 0) F(,) G(-1, -5)	E(,) F(,) G(-1, -3)	E(,) F(,) G(,)	E(,) F(,) G(,)
Trapezoid	K(,) L(6, -1) M(8, -1) N(,)	K(,) L(,) M(,) N(-1, 1)	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)

Table Top Transformations

Chart 2 (TTT-2)

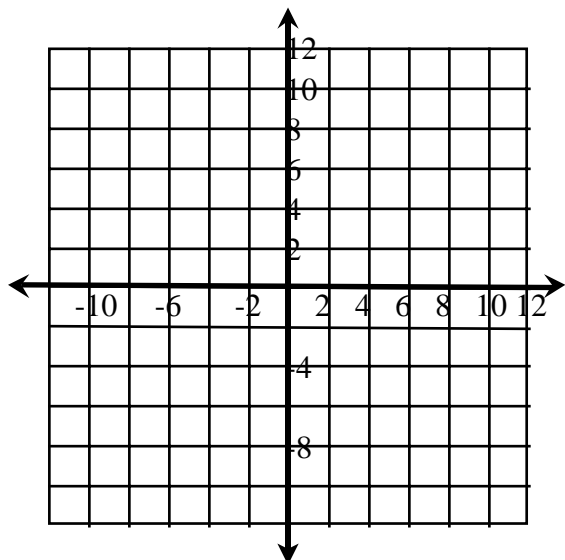
Record coordinates of original position of your choice, then the coordinates of the transformed shape. The rotation is 180° clockwise about the origin. The reflection is about the x-axis.

Shape	Original Position	Translate	Rotate	Reflect
Rectangle	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)
Right Triangle	H(,) I(,) J(,)	H(,) I(,) J(,)	H(,) I(,) J(,)	H(,) I(,) J(,)
Isosceles Triangle	E(,) F(,) G(,)	E(,) F(,) G(,)	E(,) F(,) G(,)	E(,) F(,) G(,)
Trapezoid	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)

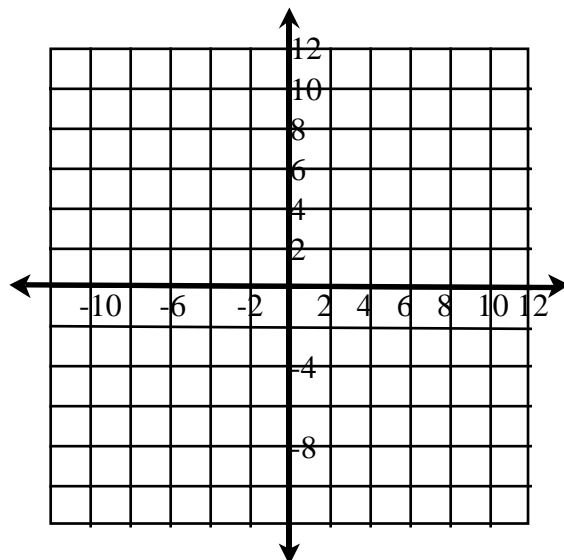
Table Top Transformations

Grid Recording Sheet

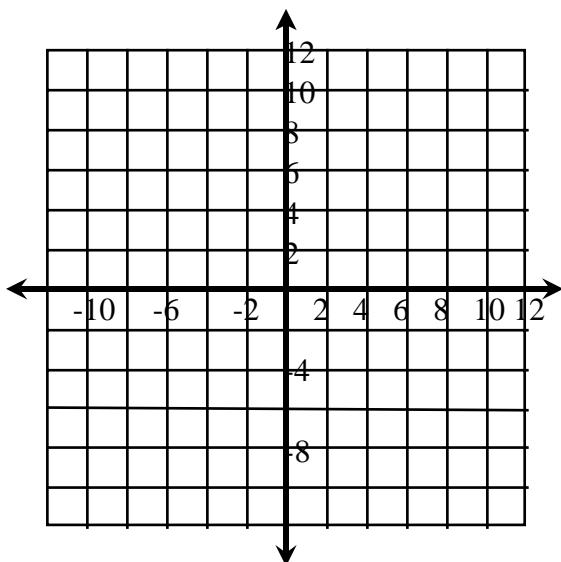
Polygon _____



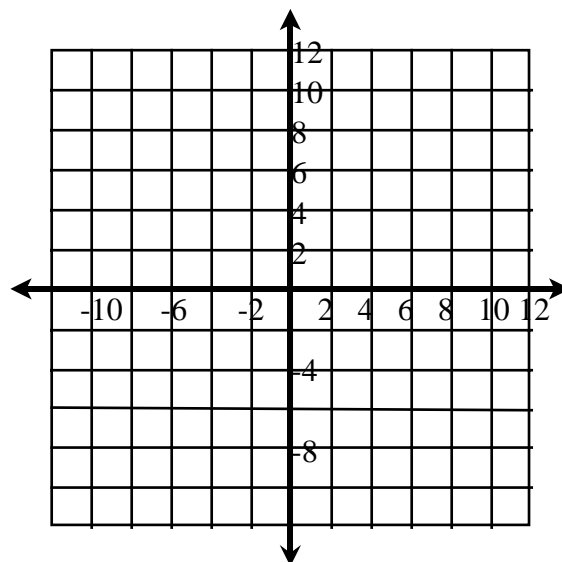
Original
Position



Translation



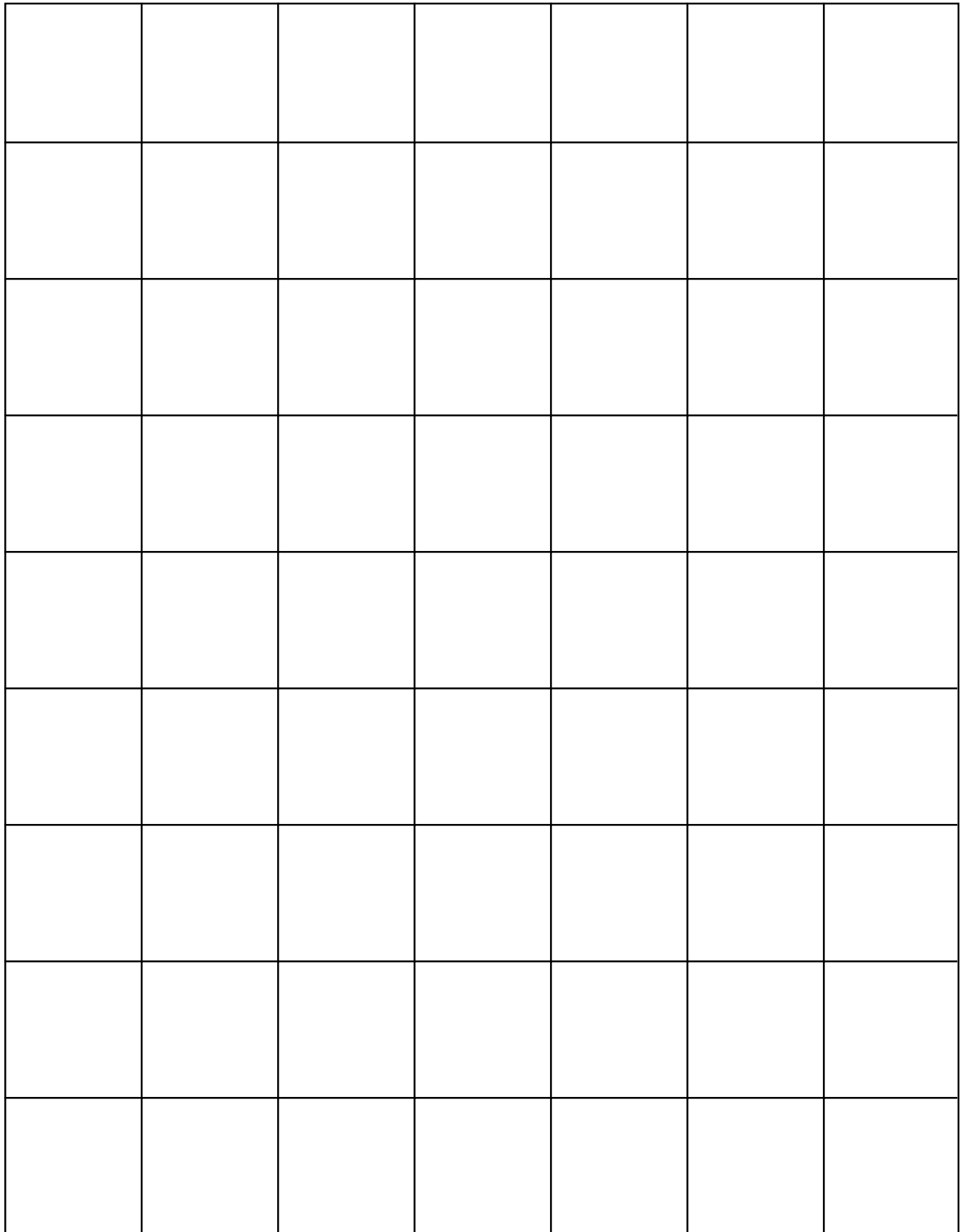
Rotation



Reflection

Name _____ Date _____

One Inch Square Grid Paper



Coordinating Change

- I. Plot the following points and draw the polygon, ABCD.
 A(0,0), B(0,5), C(5,5), D(5,0)
 What is the shape of ABCD? _____
 What is the perimeter of ABCD? _____
 What is the area of ABCD? _____
- II. On a new grid, draw a new shape. Use the same coordinates as ABCD, but change each x -value to 3 times the original value. Keep the y -values the same.
 A'(,), B'(,), C'(,), D'(,)
 How does the shape of A'B'C'D' compare to the shape of ABCD?
 What is the new perimeter? _____
 What is the new area? _____
- III. On a new grid, draw a new shape. Use the same coordinates as ABCD, but change each y -value to 2 times the original value. Keep the x -values the same.
 A''(,), B''(,), C''(,), D''(,)
 How does the shape of A''B''C''D'' compare to the shape of ABCD?
 What is the new perimeter? _____
 What is the new area? _____
- IV. On a new grid, draw a new shape. Use the same coordinates as ABCD, but change each x -value AND each y -value to 2 times the original value.
 A'''(,), B'''(,), C'''(,), D'''(,)
 How does the shape of A'''B'''C'''D''' compare to the shape of ABCD?
 What is the new perimeter? _____
 What is the new area? _____

Coordinating Change (cont.)

V. Plot the following points and draw the polygon, ABCDEFGH.

A(0,0), B(6,0), C(6,8), D(4,8), E(4,4), F(2,4), G(2,8), H(0,8)

What is the shape of ABCDEFGH? _____

What is the perimeter of the polygon? _____

What is the area of the polygon? _____

VI. On a new grid, draw a new shape. Use the same coordinates as ABCDEFGH, but change each x -value to 3 times the original value. Keep the y -values the same.

A' (,), B' (,), C' (,), D' (,), E' (,), F' (,), G' (,),
 H' (,)

How does the shape of the new polygon compare with the original?

What is the new perimeter? _____

What is the new area? _____

VII. On a new grid, draw a new shape. Use the same coordinates as ABCDEFGH, but change each x -value to 3 times the original value. Keep the y -values the same.

A'' (,), B'' (,), C'' (,), D'' (,), E'' (,), F'' (,), G'' (,),
 H'' (,)

How does the shape of the new polygon compare with the original?

What is the new perimeter? _____

What is the new area? _____

VIII. On a new grid, draw a new shape. Use the same coordinates as ABCDEFGH, but change each x -value AND each y -value to 2 times the original value.

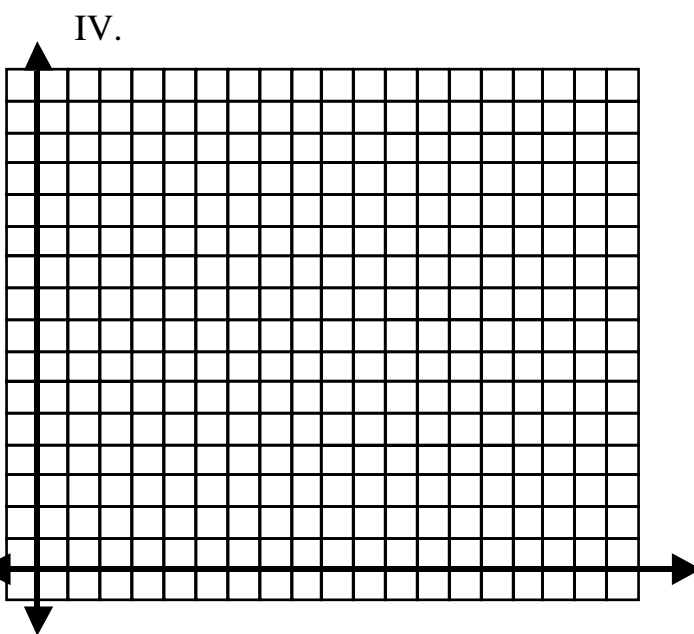
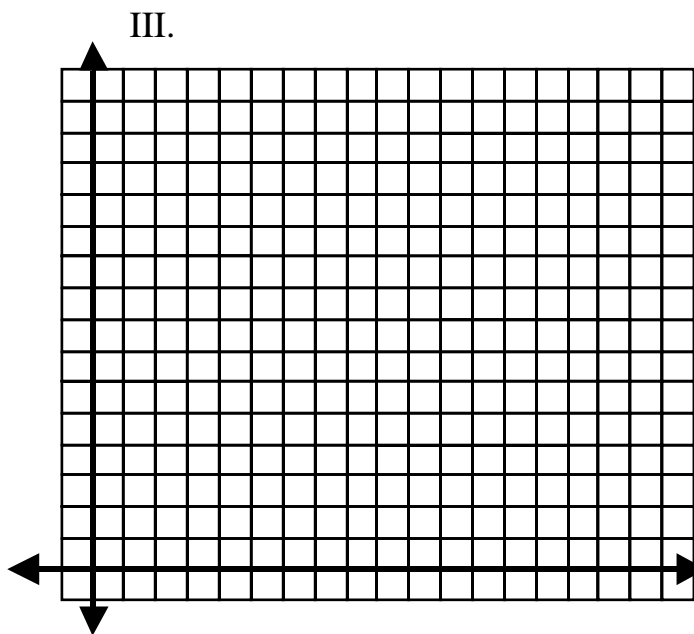
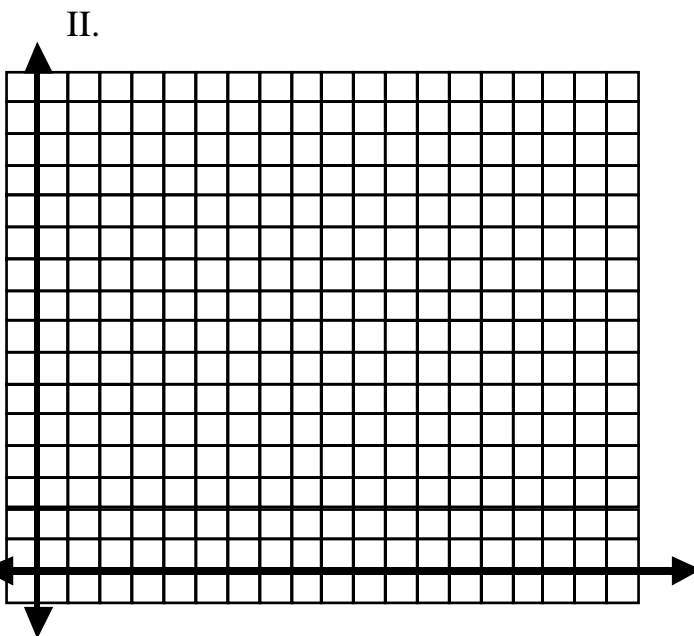
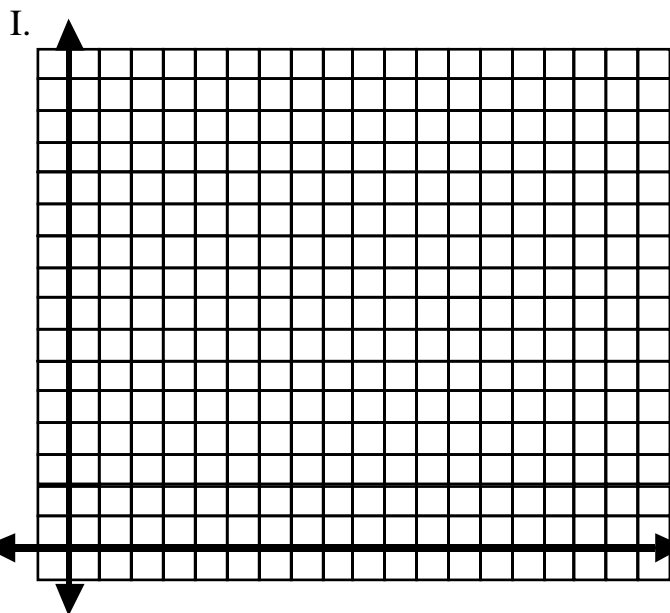
A''' (,), B''' (,), C''' (,), D''' (,), E''' (,), F''' (,),
 G''' (,), H''' (,)

How does the shape of the new polygon compare with the original?

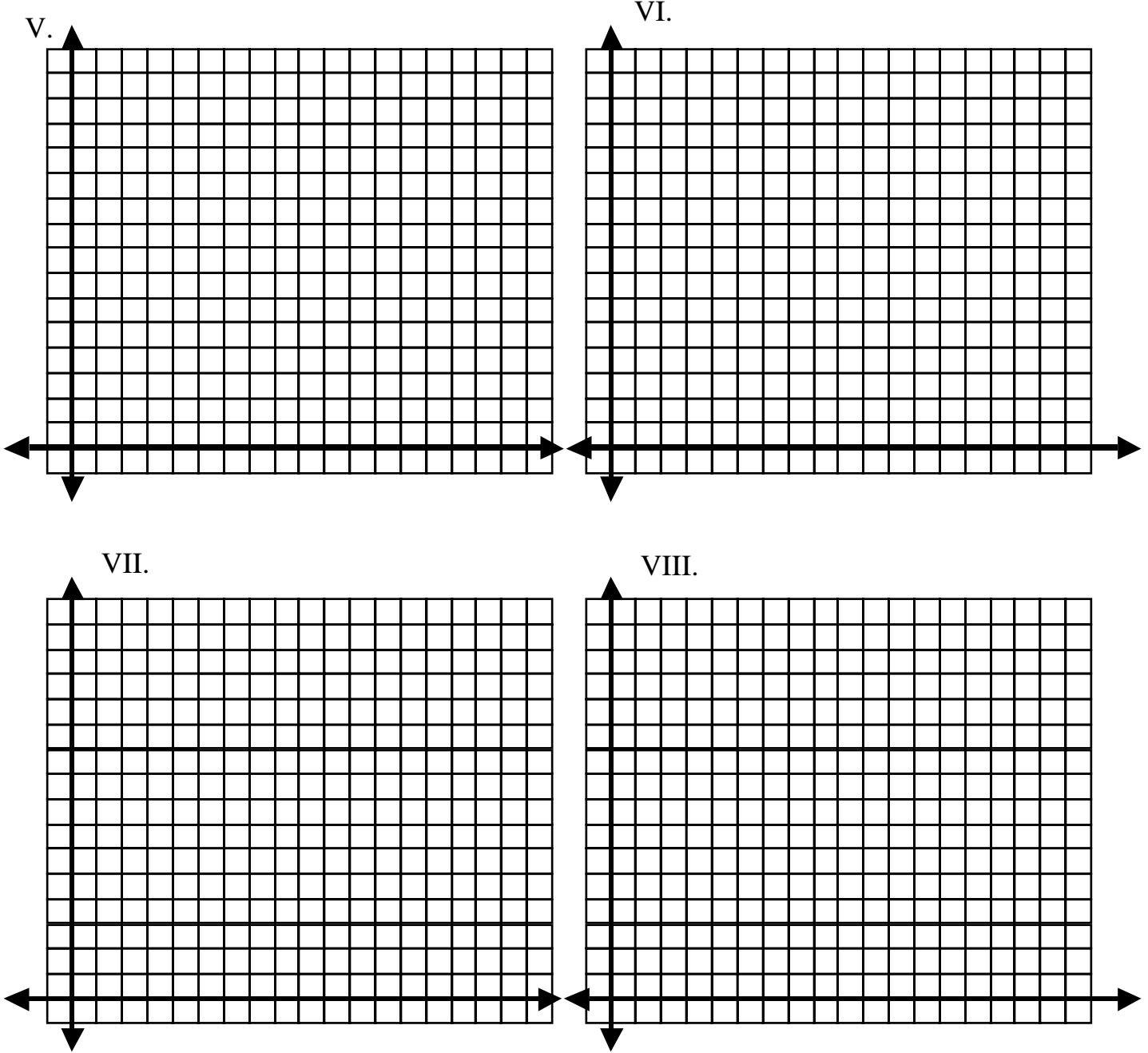
What is the new perimeter? _____

What is the new area? _____

Coordinating Change – Grid Sheet



Coordinating Change – Grid Sheet



IX. Suppose a regular pentagon is drawn so that the area is 20 square inches. An enlargement is made with each side of the new figure being twice the length of each side of the original. Predict the area of the enlarged pentagon.
