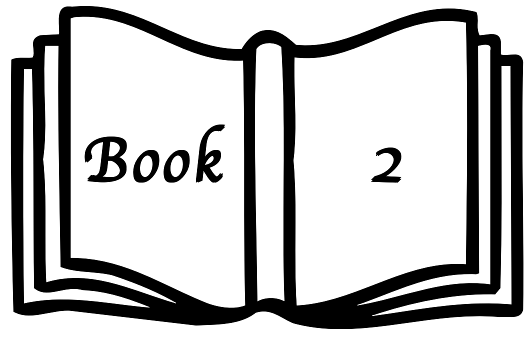
Grade Seven

Classroom

Strategies



The learner will understand and compute with real numbers.

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1.01 Write whole numbers in scientific notation; convert scientific notation to standard form; investigate the uses of scientific notation.

Notes and textbook references

A. **Scientific Notation Square Puzzle** (Blackline Master I -1) This is intended to be a cooperative activity in which students assemble 16 small squares to form a larger square. Edges of the puzzle pieces should match a number in scientific notation to its decimal equivalent. Students will benefit from listening to each other discuss strategies for solving the puzzle.

B. **Mathematical Message - I** (Blackline Master I - 2) Students will convert each number from exponential notation into decimal notation. When the numbers are placed in numerical order, a message is spelled out.

C. **Population Study** – Integrating mathematics with other subjects is one way to help students understand the importance of the math they study. In the 7th grade, students are studying African and Asian countries. Have students find the populations of each country in Africa or Asia. They should round each number to the nearest million and then write in scientific notation. Populations may be found in many reference books including almanacs. A good website for such information is the CIA World Factbook located at this website: http://www.odci.gov/cia/publications/factbook/

D. **Scientific Notation Web Pages** — The following list of websites deal with the topic of scientific notation. Most allow students to practice converting decimal numbers to scientific notation.
http://www.nyu.edu/pages/mathmol/textbook/scinot.html,
http://janus.astro.umd.edu/astro/scinote.html,
http://www.ieer.org/clssroom/scinote.html,
http://science.widener.edu/svb/tutorial/scinot.html,
http://cap.epsb.ca/math14_Jim/math7/strand1/1103.htm,

Notes and textbook references

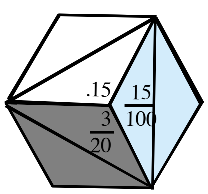
E. **Powers of Ten –** A book and short video entitled ***Powers of Ten*** was written in 1977 by Charles and Ray Eames. The Powers of Ten website, http://www.powersof10.com/, has many activities related to the topic and ordering information for an interactive CD. The essence of the film shows a picture of a person lying on a blanket which represents one square meter. Each picture following zooms out or in by a power of ten.

1.02 Compare and order rational numbers.

The concept of rational numbers is one that students have difficulty mastering. Using activities that promote peer communication allows students to develop and confirm their own understanding of fractions and decimals.

A. **Mathematical Message II** (Blackline Master I - 3) Students place the rational numbers in numerical order. When the numbers are in order, a message is spelled out. Students may work alone or in pairs. The decimal column on the blackline master suggests that changing fractions to decimals is one way to compare fractions. Encourage students to share other strategies as well.

B. **Spaceship Shape Up** (Blackline Masters I - 4 and I - 5) Materials needed: scissors, glue sticks, and blackline masters for each group. Students work in groups of three to complete the design of a space ship by filling in equivalent fractions and decimals.



A dark, light, and medium-shaded rhombus will be needed to fill in each hexagon. Equivalent fractions will meet where the rhombi meet. Each hexagon should be filled in with the dark rhombus at the bottom, light rhombus at the top, and medium-shaded rhombus at the right side as shown in the diagram here. A practical suggestion is to let one student in each group be responsible for pieces with the same shading.

C. **Rational Race** (Blackline Masters I - 6 through I - 9) Materials needed: Rational number cards, playing mat, pawns for each team, paper clip spinner. Students compete in teams to complete a racetrack. Rational number cards are divided between two teams. On a given turn, the spinner is spun to determine a target number. Then each team turns over a card. The team whose card shows a rational number closest to the target advances one square. If the team card shows exactly the target number, the team advances three squares. If the two teams are equidistant from the target number, a new target number is spun until one of the cards showing is a clear winner. The first team to reach finish is the winner.

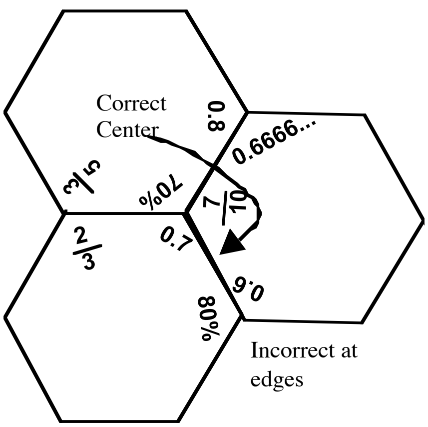
While playing this game, the students will have an opportunity to challenge each other and discuss strategies for comparing rational numbers. Communication about math concepts helps students develop and confirm their own understanding of these topics.

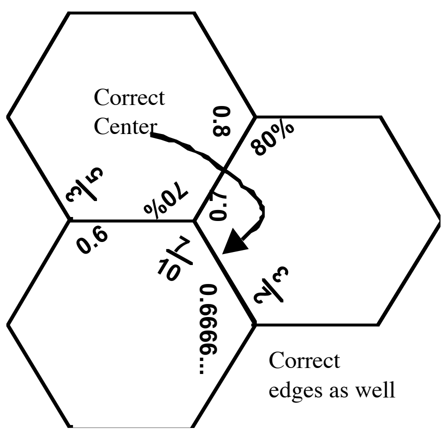
Notes and textbook references

D. Fractions, Decimals and Percent Hexagon Puzzle

(Blackline Master I - 10) Students work in small groups to assemble a puzzle with hexagonal pieces. Where three hexagonal corners meet, the pieces will show a fraction, decimal, and percentage that are equivalent. Teachers may want to print a picture on the back of the hexagon puzzle so that students can tape the pieces together and turn the puzzle over to check the picture to verify the puzzle was put together accurately.

Students should be warned that an arrangement of hexagons might have three equivalent numbers at the center, but it could still have incorrect matches at the edges as shown below. This is corrected by exchanging the position of two of the hexagons.





E. Order-Up (Blackline Masters I - 7 through I - 9)

Materials needed: rational number cards. Two teams compete in this game. The cards are shuffled, and five cards are dealt to each team. The team places the cards in order as dealt from left to right (the order battery). The remaining cards are placed in a stack face down (the draw pile). The top card is turned face up beside the draw pile (the discard pile).

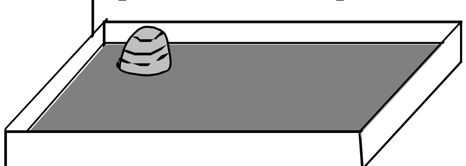
Notes and textbook references

The object of the game is to get five cards in the order battery in numerical order from smallest to largest. On a team's turn, the player can choose to take the top card from the draw pile, or the top card from the discard pile. They then decide if they wish to replace a card in the order battery with the newly drawn card. By replacing, the students are working toward getting five cards in numerical order. If they choose to replace, the card removed from the battery becomes the top card on the discard pile. If they wish, they may keep the battery as is, and place the newly drawn card on the discard pile. If the draw pile is exhausted, the discard pile is reshuffled and turned over to refill the draw pile.

F. **Recipe Workout** (Blackline Master I - 11) Students complete a chart to find quantities needed to increase or decrease the number of servings that a recipe will make. This activity shows a need to multiply and divide fractions that connects to a student's real world experiences. Patterns found in the rows and columns of the chart can help students better understand why the fraction algorithms work the way they do. The teacher may substitute any recipe that promotes interest.

1.03 Model addition, subtraction, multiplication, and division of integers; record.

A. **Modeling Signed Numbers with Heaps and Holes** This activity is based on a few lines from the movie, ***Stand and Deliver***. In the movie Jaime Escalante is trying to get his students to understand how negative numbers work by filling in holes in the sand. Explain to your students that +1 is like a pile (or heap) of sand on a level beach. A hole of equivalent size dug into the beach represents -1. This model explains positives as a surplus and negatives as a deficit.



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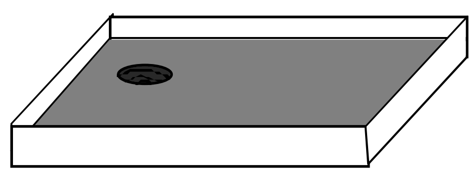
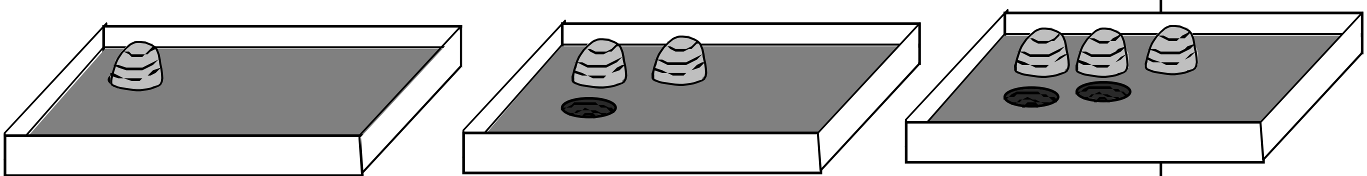


-1

First, convince your students that there are many ways to model zero. Two are shown here.

Notes and textbook references

Also show them various ways to model other numbers such as (+1) and (-1).



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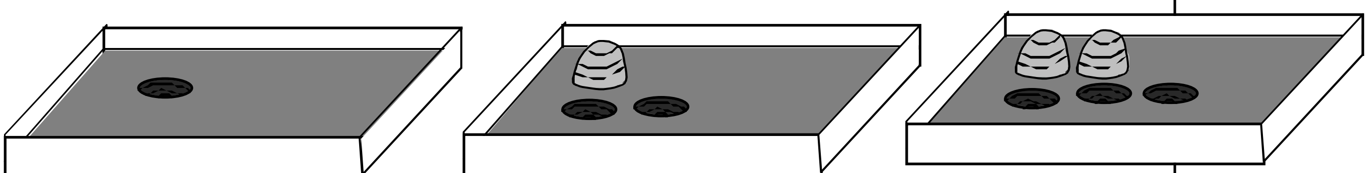
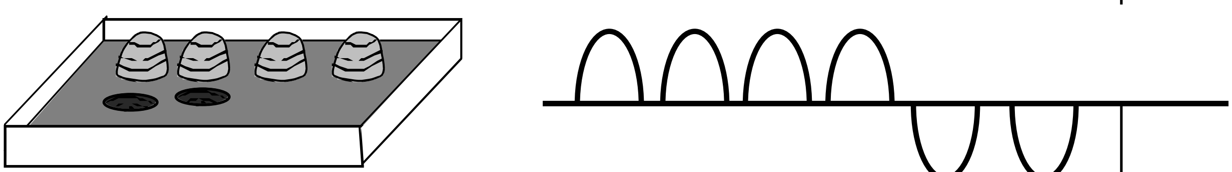
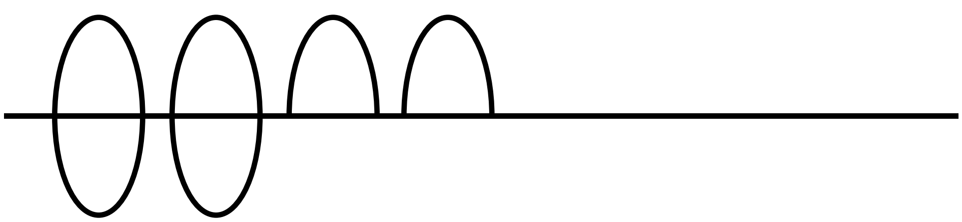


-1 -1 -1

The following illustration shows the addition of 4 + (-2). To simplify the drawing, a flat line can represent zero. Half-ovals above the line represent positive numbers (heaps) and half-ovals below the line represent negatives (holes). Here is the line drawing for 4 + (-2).



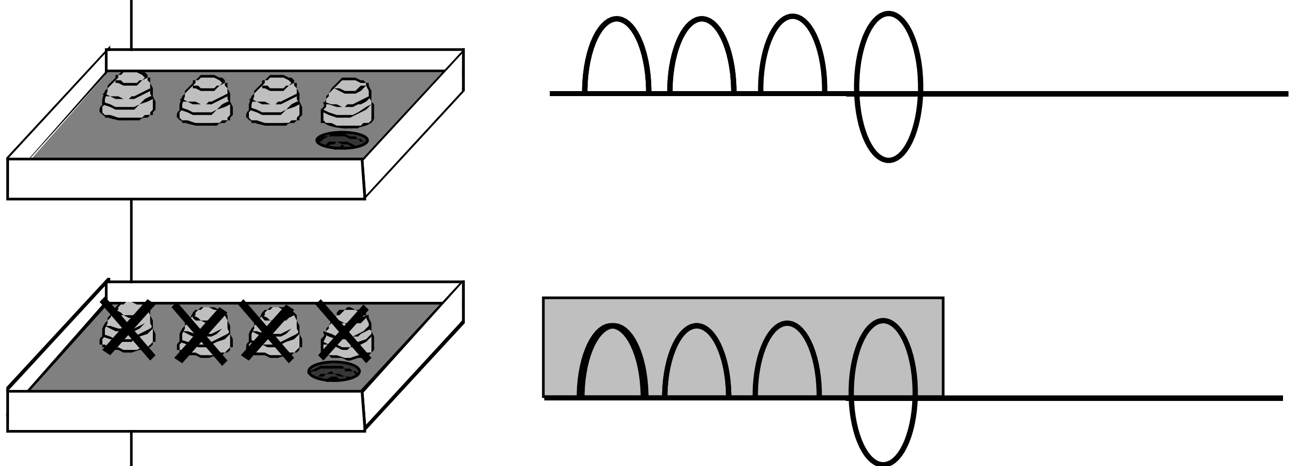
When the bottom and top ovals are lined up, a positive and a negative form something that looks like zero, and the result is displayed more clearly.



4 + (-2) = 2

Notes and textbook references

There aren’t enough heaps to take away 4, so we remedy the situation by adding a zero. The total hasn’t changed since we added a (+1) and a (-1), but now we can take away four.

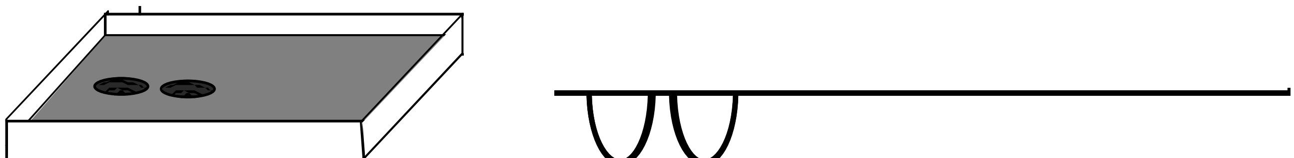


3 – 4 = -1

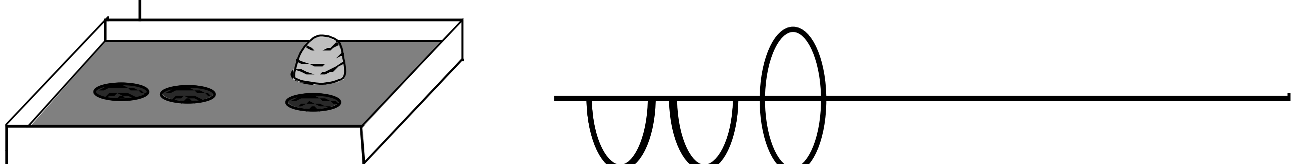
More subtraction examples:

-2 - 1

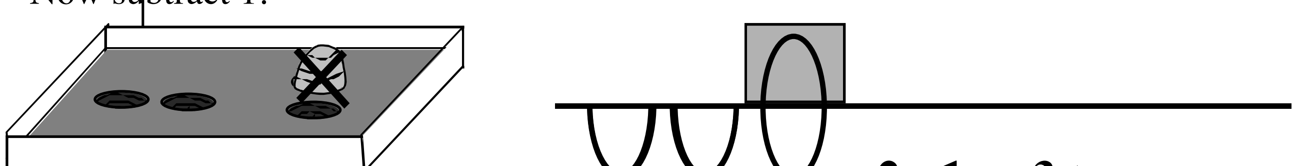
Start with -2.



There is no positive one to subtract, so we add a Heap and a Hole pair (a zero).



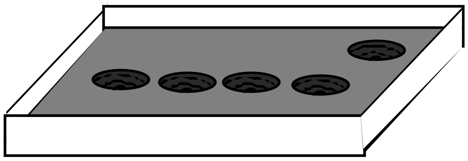
Now subtract 1.

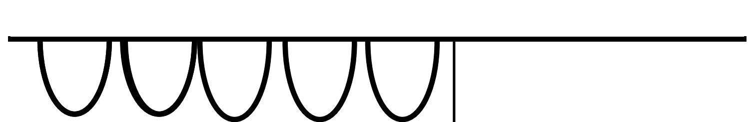


-2 - 1 = -3

Notes and textbook references

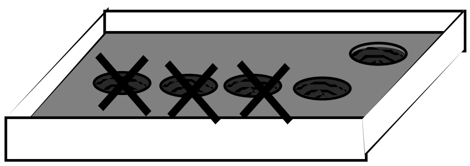
-5 - (-3) is read “Negative 5 subtract negative 3.”

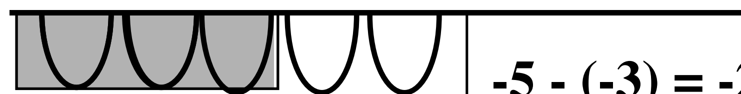




-5 **-5**

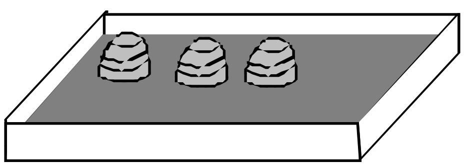
We have enough to take away (–3).



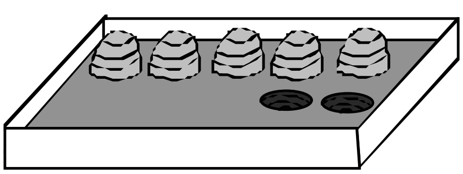


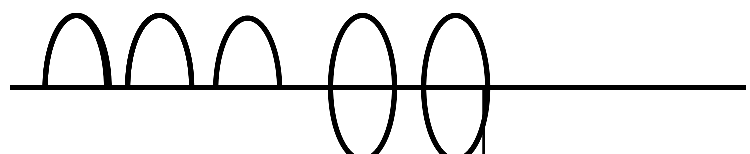
-5 - (-3) = -2

3 - (-2) **Start with 3**

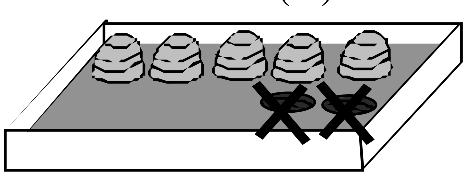


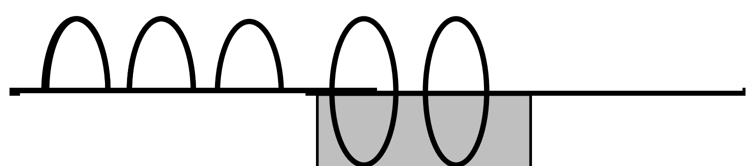
There is no (-2) to subtract so we add 2 heaps and 2 holes (2 zeros).





Now subtract (-2).

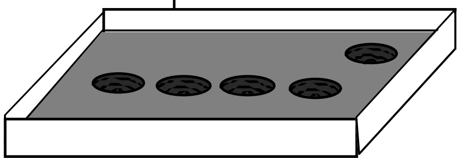


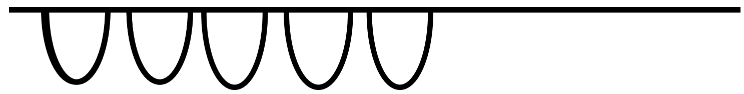


3 - (-2) = 5

Notes and textbook references

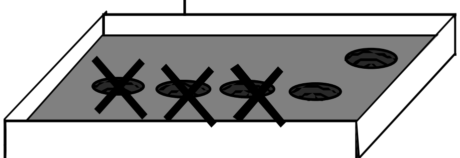
-5-(-3) is read “Negative 5 subtract -3.”

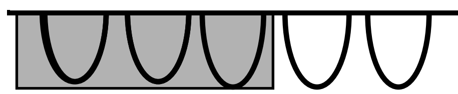




-5 **-5**

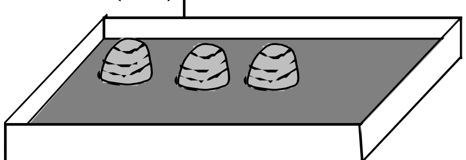
We have enough to take away –3.



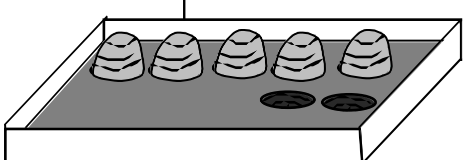


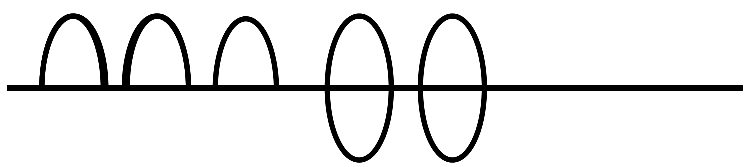
-5-(-3) = -2

3-(-2) **Start with 3**

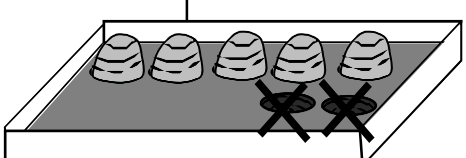


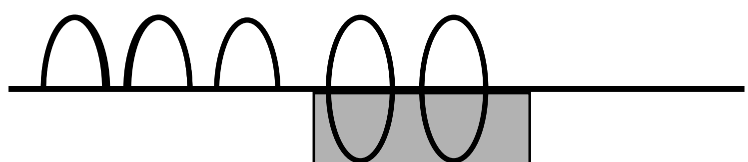
There is no –2 to subtract so we add 2 heaps and 2 holes (2 zeros).





Now subtract -2.





3-(-2) = 5

Multiplying with heaps and holes line notation is easier if you think of multiplying as repeated addition if the first factor is positive or repeated subtraction if the first factor is negative.

3 x (-2) means add in 3 sets of negative 2.

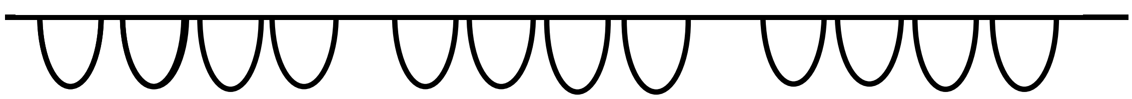
-2 x 3 means take away two sets of 3.

-2 x (-3) means take away two sets of negative 3.

Examples:

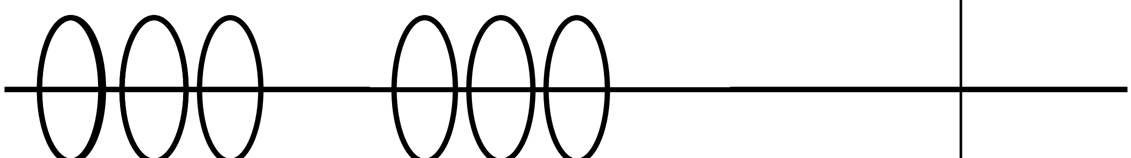
Notes and textbook references

3 x (-4) means add in 3 sets of (-4).

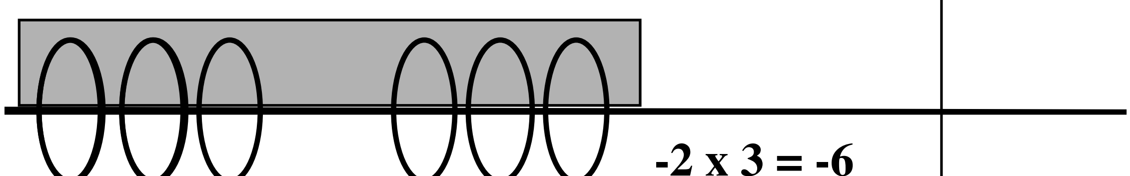


3 x (-4) = -12

-2 x 3 means take away 2 sets of 3. If we start with 0, there is no way to take away anything. But we can add additional symbols that still represent 0. Zero may be added to any number without changing the total.



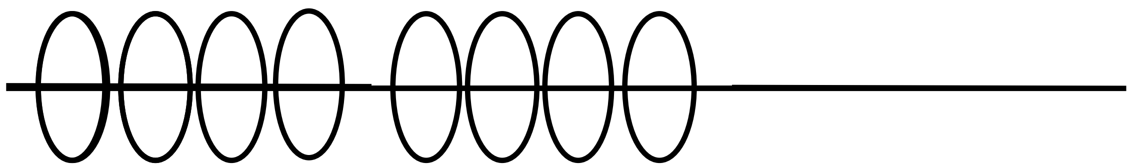
Now we can take away 2 sets of 3.



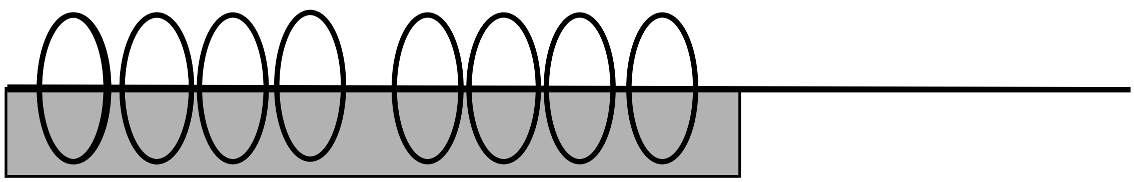
-2 x 3 = -6

Notes and textbook references

-2 x (-4) means take away 2 sets of (-4). If we start with 0, there is no way to take away anything. But we can add additional symbols that still represent 0. Zero may be added to any number without changing the total.



Now we can take away 2 sets of (-4).



-2 x (-4) = 8

B. Good Guys and Bad Guys A commonly used way of remembering rules of multiplying signed numbers is this analogy to good guys and bad guys. Think of good guys as positive, and bad guys as negative. Also, think of coming to town as positive and leaving as negative.
When good guys come to town, that's good. (Positive x positive = positive)
When good guys leave town, that's bad. (Positive x negative = negative)
When bad guys come to town, that's bad. (Negative x positive = negative)
When bad guys leave town, that's good. (Negative x negative = positive)

Another way to help students understand the concept of reversing a negative is to have them face in a positive direction. (Arbitrarily assign positive, perhaps positive is toward the door.) Now have the students turn around and walk backwards. Turning backwards and then walking backwards still results in movement in a positive direction. (Negative times a negative is positive.)

C. What about Division? -- While students are learning about fractions they surely stumble across the fact that dividing by a number is the same as multiplying by its inverse. Dividing a number by 2 is the same as multiplying it by one-half. Since reciprocals always multiply to equal one, they must have the same sign. Once we know the rules for multiplying signed numbers, the rules for dividing follow.

Notes and textbook references

1.04 Compute with integers.

Some rules and facts are easily and quickly stated but not so easily remembered and applied. It doesn't take long to state the rules for signed number arithmetic, but students need to practice with those rules so they they can become second nature to the student. Games are a pleasant way to do some needed practice. The games also provide opportunities for student communication and support.

A. Integer Addition Race (Blackline Master I - 12)
Materials needed: Pawns for the players, 2 dice of different colors, playing mat. Before starting the game, decide which color die will represent positives and which will represent negatives. On a turn, each player rolls the two dice and determines the sum of the roll. The player with the largest distance from 0 (i.e. larger absolute value) moves forward one. If a winning player rolls a sum of 5 or –5, he/she will move ahead two instead of one. If the players tie, they each roll again. The first player to reach the finish is the winner.

B. Integer Computation Square Puzzle (Blackline Master I - 13) Students reassemble 16 small squares to make a larger square. Where edges touch, an integer computation problem will match its solution. Students should work in groups of two or three for this activity. This will allow student discussion of strategies and reveal common mistakes.

For follow-up, have students make their own square puzzle. Can they make one that will stump their peers? Students should be cautioned not to use the same answer more than one time as this makes the puzzle very difficult to solve.

Notes and textbook references

No one ever learned to shoot baskets while sitting on the bench. Students can't learn problem solving by watching the teacher. We have to put the ball in their court and keep them practicing. Cooperative problem solving is one strategy to do this.

C. **Integer Computation Triangle Puzzle** (Blackline Master I - 14) Students reassemble small triangles to make the shape shown on the blackline master. Where edges touch, an integer problem will match its solution.

Students can also make their own triangle puzzles. Provide the students with blank equilateral triangles and have them arrange the triangles into a symmetric shape. At each place where edges touch, students will write in a problem on one triangle and its solution on the matching edge. Students should be cautioned not to use the same answer more than one time as this makes the puzzle very difficult to solve.

1.05 Write and solve proportions.

A. **Cooperative Problem-Solving Cards** (Blackline Masters I - 15 through I - 19) Students work in groups of two or three to solve problems. Each problem is written on three cards. Each student gets at least one of the cards and must keep it in his/her possession. The students share the information on their cards to find a solution to the problem. Each of these problems involves multiple uses of proportion. The problems were selected to review other concepts of middle school math, including money, measurement, similar figures, graph scales, and percents.

Cooperative problem-solving cards provide a way for each student to have equal input into a group solution. No one will be able to solve the problem without the information of each student - even the weakest one. Through the process of discussing the problem, the weaker student can observe the thought processes of his peers. He can also see that even the smartest students sometimes have to wrestle with a problem. No one is born with all the answers. Problem solving takes effort and persistence.

These cards are a good scaffolding tool to help weaker students continue to participate in the problem-solving process while they grow in their ability to handle problem solving efficiently. If they stop trying, they stop improving. The trick is to keep them trying.

B. **Pictorially Proportional** (Blackline Master I - 20) Students use the pictures provided to help solve proportion problems.

Visual skills are useful and even necessary in understanding many areas of mathematics. It is amazing how many students do not rely on visualization and even more shocking to find how many of them are not able to illustrate math concepts with a diagram.

The diagrams provided in this exercise can help students understand proportions in several ways. They may choose to focus on the unit concept. That is, if six pairs of jeans cost $240, then how much does one pair cost. Once they solve that problem, it is easy to answer how much 20 pairs or 120 pairs will cost. Or they may look at things another way. For example, if five waiters are needed for one row of 16 tables, how many are needed for 48 tables. Three rows would be needed for 48 tables. Each row needs five waiters.

Use a variety of methods to help students develop their understanding. Diagrams and discussion are useful means to that end.

1.06 Estimate and solve problems using ratio, proportion and percent including discounts, taxes, commissions, and simple interest.

A. **Thousand Mile Race** (Blackline Master I - 21 through I - 27) This is a classroom game played with two or three teams. The purpose of the game is to allow students to practice their mental math skills as they relate to percent, percent of increase, and percent of decrease.

Materials: A transparency of the playing mat, transparencies of the game cards which have been cut apart and placed in a paper bag.
Object of the game: To be the first team to reach 1000 miles exactly.

The cards: The deck has mile cards worth 50, 100, 150 or 200 miles. There are also GO, STOP, and CHASE cards. GO cards start the teams rolling after they have been stopped or put in a chase situation. STOP and CHASE cards are played by one team against an opponent to slow down their trip.
Instructions: Divide the class into three teams. Place three cards on the playing mat. On a team's turn, they may choose one of the cards displayed on the mat as cards in play. If they choose a mile card, the team must give the correct answer in order to gain the mile points. The points are recorded in the top rectangle on the mat.

If a STOP or CHASE card is showing, the team may choose to use one of these to play against an opposing team. If a team has a STOP card played on it, they may not gain more points until they find a GO card to get them rolling again. If a team has a CHASE card played against it, they may only use the 150 or 200 mile point cards until they find a GO card to remedy the CHASE situation. If team 1 decides to play a hazard card (STOP or CHASE) on team 2, the hazard card is displayed on the playing mat in the

Notes and textbook references

High school teachers often complain that students don't have sufficient math sense. They are often equating math sense with a student's ability to work rational number problems without a calculator. In adult life students will usually have access to a calculator, but will often rely on mental computation and estimation. Math sense, confidence and competence can be supported through encouraging students to use mental math.

Notes and textbook references

lowest rectangle under the corresponding team. It is removed when a GO card is used.
If a GO card is chosen on a team's turn, they may immediately play it to remove a hazard, or they may stockpile it to use at a later time. If none of the three cards can be used by a team, then the team has to pass its turn with no play made. After a team has chosen a card to play, the leader removes that card and replaces it with a new one drawn from the bag so that each team playing has three cards to choose from.

B. **Rational Review Triangle Puzzle** (Blackline Master I - 28)
Students work together to assemble small triangles into the shape shown. Where edges meet, a rational number problem will match its answer. The problems involved include arithmetic with fractions and percent of increase and decrease.

These puzzles not only allow students to work together to share strategies, but they also provide mental math practice. Students who need a calculator should be allowed to use one, however; the problems can all be worked using mental techniques.

1.07 Use geometric models to develop the meaning of the square of a number and its positive square root; investigate and estimate square root, checking the results with a calculator.

A. **Square Root Hexagon Puzzle** (Blackline Master I - 29)
The meaning of square root is inextricably linked with the solution to equations such as $x^2 = 4$ and $x^2 = 7$. Students will reassemble hexagonal pieces so that an equation, its radical solution, and it's decimal solution meet. Students should be advised that the matching hexagons should be aligned so that all touching corners have equivalent expressions. An example to illustrate this is shown in activity 1.02 D.

B. **Square Shake** (Blackline Master I - 30) Examples are given to help students understand the relationship between a number and its positive square root. Students are asked to estimate square roots of numbers using the perfect squares close to them.

1.08 Analyze and select appropriate operations, models, strategies and methods to solve a variety of multi- step problems using positive rational numbers, integers, and their inverses. Use calculators and computers where appropriate.

Notes and textbook references

A. **Draw a Picture** — Have students draw a picture of a word problem. They will often read the problem to find more detail if the assignment is to draw it instead of solve it. After the drawing has been made, have the students show where every number in the problem belongs. Prices go on price tags, distances go on map diagrams, etc. After developing the understanding of the problem required to complete these steps, the students will be better prepared to solve it.

B. **What's Up First?** – Have students read a problem silently and then use think-pair-share to help them determine what to do first. Think-pair-share creates an environment in which every student is expected to think. There is no waiting for the "smart kid" to call out the answer, and every student will be expected to share his ideas. This technique emphasizes the importance of the process in problem solving over the value of getting an answer.

First, the entire class is asked to think silently for a moment about what the first step will be in solving the problem. Next, the students speak softly in pairs to discuss their plans and see whether they agree or can come to an agreement about what a good first step will be. Finally, the pair shares their plan with another pair for more feedback. The group then solves the problem.

C. **Math in Print** – Materials needed: Printed information involving numbers such as sales flier, transportation schedule, menu, etc. Give students the printed information. Ask the students to work in groups to prepare questions for other students. The group should write two or three questions and make them as challenging as possible. The teacher then collects all questions and prepares a handout for the students to solve in the same groups. The students get one point for each question they answer correctly, and 3 points for each time the questions they wrote stump other groups. The group with the highest score wins.

Notes and textbook references

Review – The following resources can be used to keep track of student ability with rational numbers.

A. **Rational Review - Integer Check-Up** (Blackline Master I -31)

B. **Rational Review – Ratio, Proportion, Percent** (Blackline Master I - 32)

Tips for Problem Solving in Your Class

- Set the expectation that everyone thinks! State a problem and then give everyone a moment to think about it.
- Use think-pair-share to jumpstart your students' problem-solving processes. First they think over the question, then they talk it over in pairs, then each pair shares with a larger group.
- Don't let textbooks or other published supplementary materials thwart the problem-solving process. Be wary of texts that give many drill problems with one word problem that is solved the same way as the previous problems. Also watch out for problem sets that are all basically identical.
- Incorporate group problem solving into your lessons, so students have a chance to observe their peers.
- Use problems from a variety of sources. Ask questions in a variety of ways.
- Ask a variety of questions from the same problem source data. Students begin to anticipate what a question will be without having really read the problem. Keep them flexible in their expectations.
- Expose students to problems in which the numbers they read in the problem are not necessarily the ones they will "crunch" to solve the problem. Use price lists, menus and other materials so that students will search out meaning and not just begin to crunch numbers.