

Strategies
for
Grade Three
Instruction

Acknowledgements

During the years 1990-1995, a set of resources, *Strategies for Instruction in Mathematics*, was developed for grades K - 5. These original materials were developed by a group of enthusiastic, dedicated teachers who spent countless hours brainstorming, writing, editing, and revising ideas and activities for these resources. With subsequent revisions of the *North Carolina Standard Course of Study* for Mathematics, the decision was made to revise these resources and align them with the new curriculum. A group of mathematics educators, many of whom were the original primary writers of the *Strategies*, worked diligently updating, revising and reviewing these materials to provide a teacher-friendly resource which we hope will help the teachers of North Carolina as they implement the mathematics curriculum.

A special thanks to the primary authors of these revised materials.

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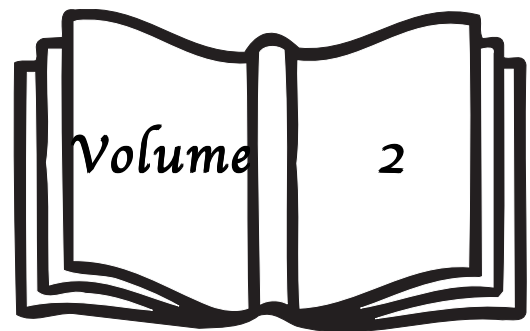
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Grade Three

Classroom

Strategies



The learner will model, identify, and compute with numbers through 9,999.

1

1.01 Develop number sense for whole numbers through 9,999.

a.) Connect model, number word, and number using a variety of representations.

A. Have children find and illustrate trivia facts which use three- and four-digit numbers. Use complete sentences to tell about the trivia illustrated. Resources: Guinness Book of Records, magazines, and newspapers.

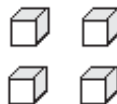
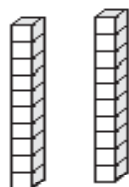
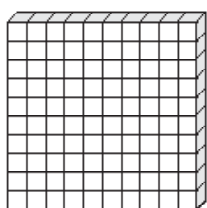
B. Create a scavenger hunt for your students using models, number words, and representations of numbers such as:

• Find the page in your social studies book that equals 30×5 . Record the first word on this page.

• Find the page in your reading book that is equal to the value of four dimes. Record the last word on that page.

• Find page two hundred forty-two in your dictionary. What are the guide words for this page?

• Find the page in your science book that is represented by



What is the topic on this page?

Notes and textbook references

Most students do not have a solid conceptual understanding of place value as they enter grade 3.

Teachers will want to plan lessons which focus on ones, tens, and hundreds as well as moving into thousands.

Students might be encouraged to brainstorm a list of titles (poems, books, songs, movies, etc.) that contain some kind of number. These titles would make good "Equation Analysis Test" items.

Three Little Pigs ...
3 = L.P.

Snow White and the
Seven Dwarfs ...
8 = S.W. and the 7 D.

Twelve Days of
Christmas ...
12 = D. of C.

C. Trace-a-Path. Teach the class as a group to do this activity before having students create new paths and clues. Give each child a hundred board (Blackline Master I - 2) and chips. Call out number riddles and have the children cover the answers on their boards and connect the numbers in the order they are called. For example, your path begins at three tens and two ones. The next step is to go to the number that is the value of six nickels and one penny. Then put a chip on the number that completes this pattern: 5, 10, 15, . . . ? As independent work, give the children a duplicated hundred board and a written set of trace-a-path directions. They draw the line from number to number as they answer each riddle. Children can later make their own trace-a-path riddles.

D. Have students brainstorm to list book titles, stories, films, songs, etc. which contain numbers. Work in groups or with partners. Some examples are The Three Little Pigs, 101 Dalmations, 1001 Arabian Nights, and "76 Trombones".

E. Organize a class **Adopt A Number** program (Blackline Master I - 3). This is a long-term project which can last all year. Decide upon a set of numbers to be adopted. This might be the numbers 0 through 100, or some other set. Adopt A Number might be started with one set of numbers and as the year progresses, other numbers might be added or substituted. Write these numbers on separate pieces of cardstock (about 8.5 by 11) or write each number on a file folder. Students will randomly draw these numbers until all the numbers have been "adopted". It might be helpful to organize the drawing so that each student has a mix of lower and higher numbers. Whenever a student learns something about his or her numbers, the student records the information on the back of the number card or on the inside of the file folder. There may be times when students are assigned to record certain types of information. For example, if the class has been discussing even and odd numbers, students will record this information about their numbers. If a student moves from the class, his or her numbers are put up for adoption by other class members. If a new student enters class, new numbers can be added or some students can give up a number for adoption. Every Number Is Special by H. Boyd, Dale Seymour Publications 1995, contains some interesting information about all the numbers from 0 through 100 and then some. For example, zero is the only number that doesn't have a symbol in most ancient number systems including Roman, Egyptian, Greek, Babylonian, and Chinese. Fifteen is the smallest odd composite number that is not a square. Students might be asked to write posters sharing all they have discovered about a number. Post these around the school.

F. Have students work in groups to find and highlight various numbers and number words in sections of the newspaper. For example, find prices in ads, weather maps, TV guide, and news articles. Students share the various ways these numbers are used. Extension: Have students make posters, write paragraphs, or videotape themselves using numbers in dramatized real-life situations.

G. Have children solve problems using number cards. This activity can be made appropriate for children at all levels and can be especially challenging for high-achieving math students. Work with a partner and with two sets of 0 - 9 tiles (Blackline Master I - 1) to solve puzzles like these:

- (1) Number as close as possible to 500 that is an odd number that uses no 4's or 5's.
- (2) Largest odd number possible using 4 digits but no number between 7 and 9.
- (3) Palindromic number you can make that is closest to 500, closest to 1000.
- (4) Largest 3-digit number the sum of whose digits is between 15 and 18.
- (5) Smallest 3-digit number, all digits are odd, the sum of whose digits is between 12 and 15.
- (6) Make a 3-digit odd number in which one digit is half the value of another and all digits are different. Extension: Have children write a number tile puzzle book to share with another class.

H. Have students brainstorm to list all numbers found in signs and logos (Phillips™ 66, speed limit 45, 7-11 stores, etc.). Work in groups or with partners; let each group make a booklet or poster or plan a class bulletin board.

Notes and textbook references

Plan to celebrate the 100th day of students 3rd grade careers. See Blackline Masters I - 11 and I - 12 for ideas.



b.) Build understanding of place value (ones through thousands).

A. Use base 10 blocks or other manipulatives to model the same number in a variety of ways. For example, 100 may be modeled as 1 hundred, 0 tens, 0 ones or it may be shown as 100 ones. (It could also be modeled as 8 tens and 20 ones.) Children can work in small groups to make charts of large numbers for bulletin boards. Use numbers to 9999.

B. Play *The Big Z*. Each pair of students needs a gameboard (see Blackline Masters I - 4 and I - 5), one number cube, several sets of 0 - 6 cards, or a spinner and one marker for each player. This game helps reinforce place value awareness.

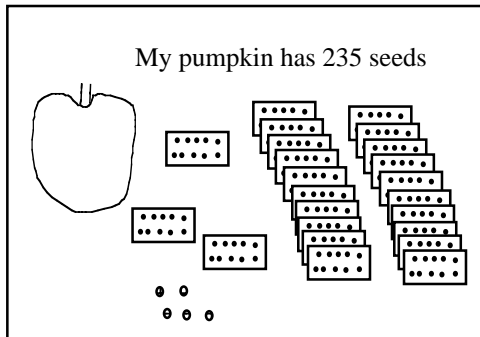
C. Give out number cards: 0 to 9, 10 to 90, 100 to 900, and 1000 to 9000. Show a 3- or 4-digit number. Have students come to the front to show its expanded form. Let different students record the expanded and standard forms on the board for all to see. When five numbers are written on the board, have students arrange them from least to greatest. See Blackline Masters I - 6 through I - 8 and I - 1.

D. Using number tiles or cards with the digits 0 to 9, have children show you a number or have a student leader give it orally. For example, The leader calls out the number 456. Each child uses tiles to show digits 4 5 6 in the correct sequence. Extensions: After the number is shown, the children state the value of each digit. For example, 5 is in the tens position and is worth 50. Have children show the number using numeration blocks or have children make another number of greater or lesser value using the same digits. See Blackline Master I - 1.

E. Use the overhead to flash flats (100's), rods (10's), and cubes (1's). Have students respond by telling what they see and what it means to them. "Smart" answers will be related to base 10 material: "I think it was 473 because I saw 4 hundreds, 7 tens and 3 ones." Be sure to stress the point that 4 hundreds = 400, etc.

An understanding of place value is basic to the development of number sense and to accurate computation.

F. Have students make collections to show groups of hundreds that are subdivided into 10's. For example: rice, bottle tops, Cheerios™, acorns, marbles, pumpkin seeds, kernels of corn. Model 3-digit numbers in many different formats, relating standard and expanded forms. Later four digit numbers are appropriate with the same activities.



G. Build an understanding of ten more or less, one hundred more or less and one thousand more or less using the calculator. Have students enter a three- or four-digit number such as 4537. Ask them to add one hundred and discuss the result. Subtract 10, what happens? Repeat the process with other numbers. How are the numbers the same? How are they different?

H. Have students enter a given number into their calculators, i.e., 453. Teacher will then ask students what they must do to change the 4 to a 3 or the 5 to a 6 or the 3 to a 2. Discuss the necessary operation. "Did you add or subtract? How did you change a number in the tens place? In the hundreds place?" Later start with a four-digit number and repeat the process.

I. Each child draws 3 boxes to represent ones, tens, and hundreds. Teacher spins a spinner (Blackline Master I - 9) and each child places that number in one box; the goal is to make the largest number possible. (When a number is placed it cannot be moved.) Continue until the three boxes are filled. Children read their numbers and compare. Variations: Have students make the smallest number, make an even or odd number, or choose a target number to try and come closest to. Spin spinner four times and have a discard box so students can decide not to use one number. Do the same activities with a fourth box for thousands.

J. Introduce **Build-and-Picture-a-Number**. A three or four-digit number is generated by spinning a spinner three (or four) times. (Blackline Master I - 9). Working in pairs, students first build this number with base ten materials, in any way they choose. For example, if 248 is the number, one partnership might build with two hundreds (flats), four tens (longs), and 8 ones (units). Another partnership might build this number with twenty-four tens and eight units. Then students are asked to “picture” this number by drawing squares for hundreds or flats, lines for tens or longs, and dots for units. In the example cited here, the first partnership would draw 24 lines and 8 dots. Students would also write $200 + 40 + 8 = 248$. The other partnership would write $240 + 8 = 248$. Then partners are asked to find a different way to build the number and repeat all the steps. They should be encouraged to find as many different ways as possible to build and picture the number. **Build-and-Picture-a-Number** could be repeated many times with different numbers. Students could generate their own numbers to explore and each partnership might be working with a different number. Be sure to plan time for partners to share their work and justify their thinking. Pictures and explanations could be posted on a bulletin board.

c.) Compare and order.

A. Select five towns or cities from your county and sequence them by:

- alphabetical order
- altitude
- population
- date of incorporation

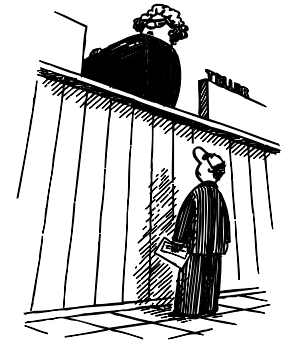
Compare and discuss the different groups.

B. Play **Order in the Courtroom!** Students will need a deck of number cards and a spinner (Blackline Masters I -27 through I - 30). In turn, a student spins the spinner and draws the appropriate number of cards from the deck. The student then orders the cards (least to greatest) and the other players agree by saying “Order in the courtroom!” Cards are returned to the deck, shuffled and the next player takes his/her turn. Points are awarded for each card ordered correctly in a successful turn.

C. Divide class into groups of four to six. Give each child an index card with a number on it. Children will put themselves in order from least to greatest according to their index cards. Trade cards with other groups and repeat the activity.

D. Each student places 0 - 9 cards (Blackline Master I - 1) face down on desk. Draw 4 cards. Using only these numerals, show all possible 4-digit numbers. Record all possibilities. Read and discuss the possibilities with the group. Repeat several times. Ask children to think of a way to organize their lists so that they know they have all of the possibilities. Have students complete the **House Numbers** worksheet. (See Blackline Master I - 10.) It may take more than one class to complete all of the tasks, but students will have a design to take home and the class will have a mathematics and social studies bulletin board.

Notes and textbook references



E. Use a version of “**Follow the Clues with Tiles©**” by Marcy Cook. Students work in groups of four. Each student uses his/her own set of number tiles, square-inch tiles numbered 0 through 9 (Blackline Master I - 20). Each student in the group receives a direction card instructing her/him to build a number according to a given property. These properties could relate to comparing and ordering numbers less than 10,000. After each student in the group has built this number, the group works together to build a fifth number which fits all four properties. While building this final number, students may use any of the number tiles; they are not limited to the ones already placed while building individually. Here is an example:

Build a number greater than 3640.

Build a number less than 3754.

Build a number between 1360 and 1370.

Build a number less than 4366.

Much more could be included in these cooperative problems in order to address additional objectives and skills. See Blackline Masters I - 14 through I - 18.

F. Using number tiles or place value mats and counters, have children show the number you call out. For example: Show me a number that is less than 4620. Show me a number that is greater than 1270 but less than 1503. Show me an odd number that is greater than 7628 but less than 8889. “Show me the number that is one less than 180.” Notice that some “show me” statements have one correct answer, while others are more open-ended.

G. Using a map, find distances between towns. Have students make up stories about trips to and from three or four towns. Compare the total miles traveled. Let children talk about how long they think they need for a trip. Be sure students explain their reasoning orally.

H. Make a number “clothesline”, marking 0, 50, 100, 150, 200, etc. with large cards (Blackline Masters I - 6 through I - 8, and I - 54) pinned to the line. Have students fasten smaller cards with number such as 294, 43, 128, 586, 72, 423 in the appropriate places (Blackline Master I - 13). Use the same idea for helping children learn to find the nearest 10 by pinning cards marked 0 to 100 on the line. Be sure to leave spaces to add cards. (*Numbers could be written on clothing shapes to make a colorful number clothesline.*)

I. Play **Guess Your Number**. Write 3- or 4-digit numbers on cards, one card per student. Put one card on each child's back, making certain the student does not see the number. Children move around the room, asking questions about the number. Questions must be answered "yes" or "no". You will be able to learn a great deal by moving among the children to hear the types of questions they ask. When a student discovers the number, it can be placed on the front of the body so that the child can continue to answer questions. After playing the game, discuss with the class the kinds of questions that are helpful in trying to discover a number. Save the cards to be used another day. (This is a good rainy-day activity for controlled movement.)

J. Play a variation of the same game: **Guess My Number**. The teacher tells students, "I'm thinking of a number." The teacher can write the number on the overhead and cover it up. "Can you guess my number in less than 10 questions that can be answered yes or no?" Have students devise ways to eliminate numbers. Discuss questions that eliminated the most numbers. Gradually allow students to eliminate numbers on their own. Examples, "Is the number less than 1000? Is it more than 5,000?"

K. Line-Up! The object of this classroom game is to try to put numbers in order from least to greatest. Each child will put 5 blank lines on his paper. The teacher rolls four number cubes which represent a four-digit number. The children may select the order for these digits (For example, If the teacher rolls a 2, 6, 2 and a 4, the student may choose 6242 or 2426) and put the number on the blank where they think it will fall in sequence at the end of 5 rolls. Winners are all students whose numbers are in a correct sequence. Note which children quickly realize that 6666 is the highest possible number and that 1111 is the lowest. Because numbers may not be moved once they are written down, children who know how to sequence numbers correctly may not win because of the placement they have used.

A diagram of a rectangular box with a double border. Inside the box, there are five horizontal lines. The second line from the bottom is filled with the number 2426.

1.02 *Develop fluency with multi-digit addition and subtraction through 9,999 using:*

a.) *Strategies for adding and subtracting numbers.*

When you are subtracting, multiples of ten are friendly because they are willing to “give away one.” Which of these is easier to compute?

$$\begin{array}{r} 50 \\ -37 \\ \hline \end{array} \quad \begin{array}{r} 49 \\ -37 \\ \hline \end{array}$$

Take one away from the 50 while computing and then give it back to the difference.

$$50 = 49 + 1$$

$$49 = 50 - 1$$

Use the idea to make situations such as this easier to resolve:

$$500 - 389 =$$

A. Talk about **Friendly Numbers**. The purpose of this activity is to help children identify numbers that make “adding to” and “skip-counting from” another number easier. For example, multiples of 10 are friendly numbers because they are easy to use in operations.

$$67 + 29$$

$$79 + 14$$

$$29 + 41$$

$$67 + 30$$

$$80 + 14$$

$$30 + 40$$

When adding $49 + 56$, ask children what (multiple of) ten is 49 closest to. Fifty is more than 49, so children would add $50 + 56$ and then subtract from the sum. (Be certain the children understand that they “gave” 49 one more, so they will have to reclaim the one). Many children will need to do this activity at a concrete level before working with symbols alone. Have children solve the same addition situation in other ways, including the traditional “regrouping”.

B. Demonstrate the strategy of re-naming a multi-zeroed number by subtracting one from the minuend (3000) and the subtrahend (1278) before subtracting.

$$\begin{array}{r} 3000 \\ -1278 \\ \hline \end{array} \quad \begin{array}{r} 2999 \\ -1277 \\ \hline \end{array}$$

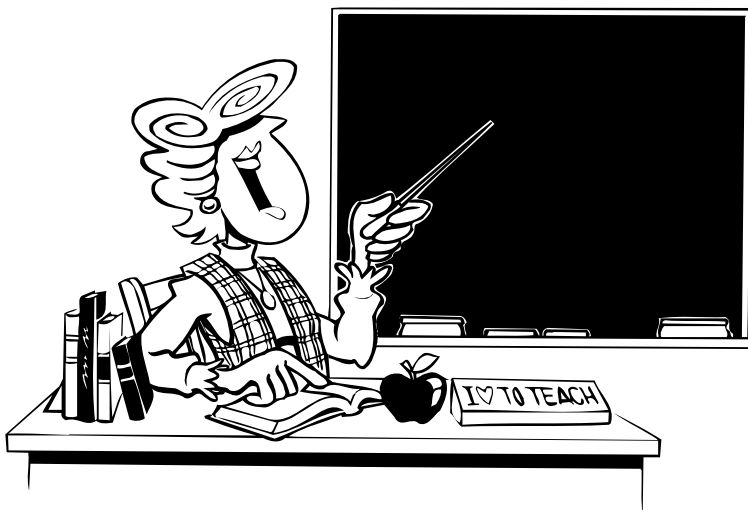
C. When summing three or more addends, model adding benchmark facts such as doubles, combinations of ten, doubles plus one, or doubles minus one to increase fluency in finding sums.

Notes and textbook references

$$\begin{array}{l} 3 + 1 + 7 = \\ \swarrow \quad \searrow \\ 10 + 1 = 11 \end{array}$$

$$\begin{array}{l} 7 + 2 + 7 = \\ \swarrow \quad \searrow \\ 14 + 2 = 16 \end{array}$$

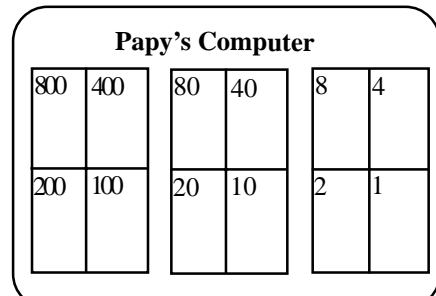
$$7 + 8 \quad \text{think} \quad 7 + 7 = 14 \quad \text{so} \quad 7 + 8 = 15$$



Notes and textbook references

D. Use a calculator to demonstrate repeated subtraction. Enter $39 - 3 = = = \dots$ etc. Count how many times you subtract until you reach 0.

E. Use Papy's Computer to model numbers and compute in non-traditional ways. See the discussion in the Blackline Masters I - 31 through I -35.



b.) Estimation of sums and differences in appropriate situations.

Notes and textbook references

A. Set up two or more estimation jars. Have contents of each jar be similar such as two or more different kinds of candies. Have students estimate total candies in jars. Have the students estimate how many more or how many less are in each jar.

B. The book Pigs Go To Market by Amy Axelrod is about the Pig family going shopping for Halloween candy. Create questions using the middle spread of pages which details the aisle of the market complete with signs which include prices and amounts.

C. Track amounts of items that the class uses such as laps on the school track. Set a class goal and repeatedly have the students practice estimating how many more laps to reach their goal. The same procedure could be done with reading points or behavior goals.

D. Make use of sale flyers by having students plan a shopping spree with a certain amount of money. Focus on the estimate of their total instead of the actual at first. Later use pencil and paper or calculator to have students see how close they were able to get to the specified amount.

c.) Relationships between operations.

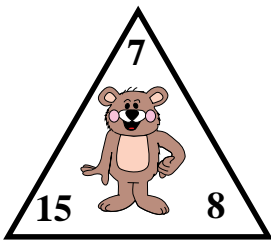
A. Write on the following on the overhead or board:

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 24^*$$

$$\underline{24^*} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

(*the total number of students present). Have the students stand up and form two groups. Fill in the blanks by writing an equation based on the number of students in each group. Have students discuss how they would fill in the subtraction equation based on the way they separated into groups. Allow students to find different combinations of groups and have them write addition and subtraction equations based on the way that they formed their groups. Have students discuss how the two equations are related. Ideas for groups: boys/girls, likes math better/likes reading better, short hair/long hair, tennis shoes/other kind of shoes, blue eyes/brown eyes, etc. After allowing the class to experience the whole class activity, allow students to work in groups of four to write more related equations about their group. Each group should be allowed time to share some of their equations with the rest of the class.

B. Each student needs a collection of small manipulatives such as beans, cubes, or colored counters. Using addition and subtract operations, have students generate number sentences that use the same three numbers. Students use the counters to create a model of the relationships. Have students illustrate the relationships and write out the number sentences using symbolic representation.



Ex. $10 + 15 = 25$
 $15 + 10 = 25$
 $25 - 10 = 15$
 $25 - 15 = 10$

Bring students together in small groups to share, and discuss the relationships discovered during independent work.

C. Use a set of triangle cards with three numbers corresponding to an addition fact. Have students draw a card then write two addition facts and two subtraction facts based on the numbers from their card. Allow students to trade cards and then compare/discuss the facts that were written. See Blackline Masters I - 57 and I - 58.

1.03 Develop fluency with multiplication from 1×1 to 12×12 and division up to two-digit by one-digit numbers using:

a.) Strategies for multiplying and dividing numbers.

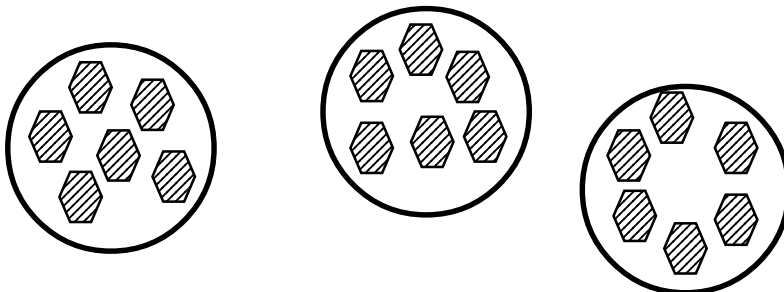
A. Use a variety of manipulatives to explore multiplication. For example, connecting cubes can be stacked in sets. (They can also be worn on fingers!) Goldfish crackers can be grouped in schools on a blue construction paper pond. Strips of equal length paper can be laid end to end. (A 2-inch strip + 2-inch strip + 2-inch strip = 6-inch strip. Glue and write the equations. $2 \text{ in.} + 2 \text{ in.} + 2 \text{ in.} = 6 \text{ in.}$ or $3 \times 2 = 6$.) Use chips or cubes in an egg carton to model multiplication. Put equal sets of seeds in little cups to show multiplication.

B. Give students construction paper circles and color tiles, beans, or other counters. Ask students to model 3×6 , by putting three circles on desk or table. On each circle, students will put 6 objects. Have students record both the multiplication act and the related addition sentence.

$$3 \times 6 = 18$$

$$6 + 6 + 6 = 18$$

Next, ask students to model 6×3 and record the multiplication and related addition sentence. Discuss the commutative property. Continue to have students model different facts including those with a factor of zero. It is also important to model facts with the factor of one.



C. Give students 24 objects such as beans. Have students sort them into 3 groups, sharing equally. Have students put them back together and sort them into other groups: 2, 4, 6, 5 (talk about “leftovers” or remainders), 8, 12.

D. Use a variety of manipulatives to explore division. Give children a train of connecting cubes that can be divided into equal groups. Put a dozen fish crackers in a blue construction paper pond. Children divide them into equal schools. Give children a 12 inch strip of paper. Children fold them into even sections, cut, and mount. Students are given a set of marshmallows and cups. They divide their marshmallows equally into their cups. Discussions should include the relationship between multiplication and division, the idea of division as repeated subtraction, and division as sharing equally. Have children begin recording while using models.

E. Use the **Division Dilemma I** and **Division Dilemma II** Blackline Masters I - 21 and I - 22 to give students many opportunities for relating manipulative experiences with record keeping.

F. Sort objects from the classroom into equal sets. Assign both sharing equally jobs (“*Divide these crayons into six groups for our tables.*”) and division by subtraction (“*How many groups of five can you make with these cubes?*”).

G. Divide the students into groups of 4. Give each group a small box of raisins. Have students share the raisins equally. Allow each group to tell how they shared the raisins. Emphasize that division is sharing equally. Discuss what they did with any extras.

H. Give students specified number of counters (for example, 35). Ask students to take out sets of five. The teacher should record the repeated solution each time a set of five is taken out. Repeat this activity with other numbers. Have children record the process.

35 ÷ 5 means
35 - 5 = 30 (1)
30 - 5 = 25 (2)
25 - 5 = 20 (3)
20 - 5 = 15 (4)
15 - 5 = 10 (5)
10 - 5 = 5 (6)
5 - 5 = 0 (7)
35 ÷ 5 = 7

I. Give all students objects such as beans or counters. Ask students to make up stories to go with numbers you give them. For example, have them tell a story and act out this situation: 20 objects and four people or 18 objects and five people.

J. Use a calculator to demonstrate repeated subtraction. Enter $39 - 3 = = = \dots$ etc. Count how many times you subtract until you reach zero.

K. Create problem situations, real or imaginary, for your class to discuss. “We are planning a Treasure Chest for a class of 25 younger students. The chest should include three small prizes for each child. Since we don’t have much money to spend we need to find the best deal. You must choose bags of toys and decide how many bags you need. Bag A has 10 toys at \$3.00 per bag, Bag B has 20 toys at \$4.00 per bag, and Bag C has 5 toys at \$1.50 per bag. What will you decide to buy and why?” Use connecting cubes and play money to explain what you decide. Remember, sometimes the “best” toys may cost more.”

L. Pour cereal into ziplock™ bags (one per group). Have students estimate how many pieces of cereal are in the bag. (Use cereal with large pieces.) Have groups count out the cereal. Ask how they could divide the cereal equally. What will they do with any remainders? Variation: Use other food packages that students may consume.

M. Set up a store using items such as teddy bear graham crackers, wrapped small candies, erasers, pencils, stickers, and so on. Have students use play money to pay for items. Assign single digit values to each item. Have children multiply to find out cost, i.e., 4 crackers at 5¢ each cost 20¢. Limit purchases to under \$1.00.

N. Students play with a partner to practice multiplication tables. For example, if practicing fives, write 5 on a piece of paper or use a number tile. In turn, students will roll (0 - 9) die. If first student rolls a 6, then the student will multiply 6×5 and record. Next, the partner will roll, multiply, and record. Person with largest number circles the product. After 10 rolls, person with most circled numbers wins game. If a 0 - 9 die is not available, draw digit cards or use a spinner.

O. Square color tiles are a useful tool for developing this objective. They can be used as a collection of objects or to make rectangular arrays. Explain and model the following activity. Roll a die, spin a spinner, or generate a number from zero to nine in some other way. The resulting number indicates how many pieces of paper to set out; rectangles about four inches by five and one half inches will work well. Then roll the die a second time. This number indicates how many square tiles are placed on each piece of paper. For example, if the first roll of the die resulted in 4 and the second roll resulted in 5, you would set out 4 pieces of paper and on each place 5 square tiles. Count the total number of tiles out loud and then write $4 \times 5 = 20$. Consider accompanying this model with a story line about how you have invented a new candy called “**Squittles**.” Once you have created some “bags” of these candies, you want to put them together into a rectangular candy bar. Model taking the square tiles and arranging them into a 4 by 5 rectangle, 4 rows of 5 squares. Record this “candy bar” by coloring a 4 by 5 array on square inch grid paper and writing $4 \times 5 = 20$. Explain that you want to keep track of the candy bags and bars in order to decide how best to package your new candy. Then distribute materials and have students continue this activity or “research” on their own. Lead class discussions related to patterns. What happens when one of the rolls is one? What about 4×5 and 5×4 ?

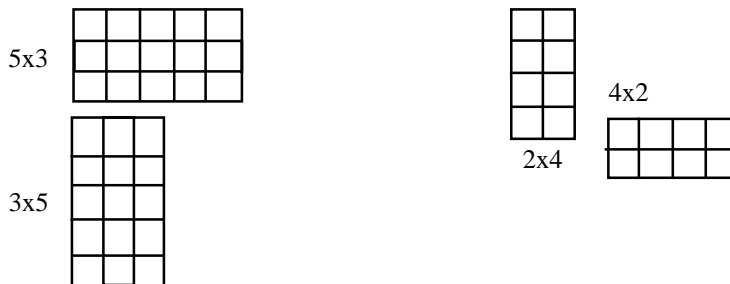
P. After some time with this “**Squittles**” activity, give 24 tiles to each small working group. Have them distribute or “divvy up” these into 3 groups, or onto 3 pieces of paper. Then ask students to write $24 \div 3 = 8$. Discuss the relationship between $3 \times 8 = 24$ and $24 \div 3 = 8$. Also discuss how this is like subtracting 3 at a time and repeating this process 8 times. Also relate this to a rectangular array of 3×8 . Consider continuing the story line about “**Squittles**” by explaining that each group of students will get a bag of 24 and their job is to divide the candies up among 1, 2, 3, 4, 5, 6, 8 and 12 students. What would happen with 7, 9, 10, 11 students?

Q. Prepare a class game for the overhead (or on a large poster). Draw or cut out pictures of toys and assign different prices to each. For example, a football might cost \$7, a baseball \$3, a bat \$5, a soccer ball \$6, a jump rope \$2, and a large rubber ball \$4. On a 5 x 5 inch grid put a variety of prices, all greater than the highest priced toy. Prices might range from \$8 to \$32. The object of the game will be for a team to cover 4 prices in a row. In order to earn the right to cover a price, a player must state what can be bought that will total that price. For example, a student could buy 3 soccer balls and a jump rope and cover \$20.

R. **Buzz** is a class game used to practice multiplication tables. You name a table you want the class to practice. For example, the five tables. Children stand next to their chairs. The first person says 1, the next 2, and so on. When it is time for the fifth person to name the number, the student must say “**Buzz.**” If the student forgets and says “five”, the student sits down. Each time a player is to say a multiple in the practice table, the student must say “**Buzz**” or be seated. Winners are those left standing at the end of the game.

S. Have students brainstorm lists and create classroom posters for things that come in 3’s, 4’s, 6’s, 7’s, and 8’s. For example: 3 Little Pigs, 3 wheels on a tricycle, 3 legs on a tripod, 3 Musketeers, 3 wishes, and so on. Ask students to use the information from the charts to write their own story problems and then work them. For example, a student might write, “The 3 Little Pigs decided to have parties to celebrate moving into their new houses. Each Little Pig invited 5 guests. How many guests came to the parties?” These problems could also be posted for everyone to solve. Allow students to use beans, connecting cubes, tiles and so on to help them solve these problems.

T. Give students squares of paper or tiles to build rectangles whose sides are the factors in multiplication problems. Children need many opportunities to make rectangles, noticing the relationship of the sides with the total number of squares used.



U. Give each student a multiplication problem such as 3 x 5 or 6 x 2. Have students create a problem or draw a picture to illustrate the situation. Discuss and share the results. Put these in a class book.

V. Have students make any design they choose on their hundred boards with chips. Then ask them to write a series of multiplication or division facts to form that picture. Direct students to exchange puzzle directions with other members of the class.

W. Have students look for the missing factor in the multiplication facts using **Cardofacts** (see Blackline Masters I -23 through I - 26). Each student will have a deck of cards containing three sets of cards numbered 1 - 9 (Blackline Master I - 1), and the group will have one set of product cards numbered 20 - 100, and a hundred board (Blackline Master I - 2).

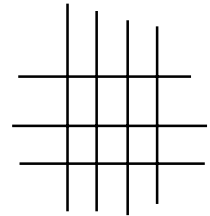
Rules: The first player will turn over a card from his/her deck, then turn over a card from the product pile. He/she must state the missing factor that will give him/her the product he/she turned over. Then have students write their initial in the square that contains the product on the hundred board. The next player takes a turn. The player to get three in a row (diagonally, across or up and down) wins. If a player cannot name the factor, he/she loses a turn and the next player turns over a card from the deck to get the product previously turned over. Play continues until someone gets the three in a row. A FREE card allows the student to use two cards from her/his pile to name a product not claimed on the hundred board.

Variations: From the product deck prime numbers may be removed. You may add or pull cards out of the players' decks. Use a multiplication table instead of a hundred board. Have the students cover or initial the missing factor instead of the product.

X. **Factofish** is a variation of Fish. Deal seven cards to each player. Students will need to get three cards to form a multiplication fact. The rules are the same as Fish. (See Blackline Masters I -23 through I - 26 and I - 1.)

Y. Play **Divide and Conquer!** Each pair of students has a deck of cards (see Blackline Masters I - 23 through I - 26), shuffled and placed face down in a pile, and a spinner. The first player spins the spinner to determine the divisor in question. Then she/he turns over the first card in the pile and must decide if the number is divisible by the number spun. A calculator should be used to check the accuracy of the response. If the student is correct, she/he earns one point. Play then moves to the next player who spins, chooses a card and repeats the process. Discards move to the bottom of the pile. First player to score ten points has conquered!

Z. “**Intersections.**” If Juan is working on 3 times 4, he draws 3 horizontal lines and then 4 vertical lines crossing the first 3. These lines represent streets crossing each other. Juan needs to determine how many intersections exist so he counts the points where the lines cross. He records this by writing $3 \times 4 = 12$. To vary this, students can create “Berry Bushes.” If Juan is still working on 3×4 , he draws a stem with 3 branches. He then draws 4 little berries on each stem, counts the berries and writes $3 \times 4 = 12$.



AA. “**Password.**” Ask each student to pick a specific fact to memorize. As students pass through the classroom door, ask them for their “password.” For example, as Juan goes out the door to lunch, you ask, “What’s the password?” and as Juan replies, “ $3 \times 4 = 12$.”

BB. Read *The Doorbell Rang* by Pat Hutchins. Have students write a number sentence for each problem.

CC. When practicing the division algorithm, have students use the divisor of 5 since they can use their knowledge of money to help them predict a reasonable answer.

DD. Read Marjorie Sharmut’s *The 329th Friend*. How many tables for three and tables for eight will Emery need for his guests?

EE. Copy a menu from a local restaurant, make a poster with fast food prices, or create an overhead which gives food prices. Have students figure the costs for food in the following situations:

Big Burger.....\$ 2.39
Chicken Delight..\$ 2.65
Baby Burger.....\$ 0.74
Golden Fries.....\$ 0.83
Large Drink.....\$ 0.99
Small Drink.....\$ 0.75

- order 3 of the same sandwich
- 2 people order the same sandwich and a large drink
- 5 people order the same size drink
- 4 orders of the same sandwich, drink and small fries

Note: Since students are not yet formally multiplying with decimals, they should decide where the decimal goes by using logical thinking and relating multiplication to repeated addition. For example, would six Baby Burgers cost \$444.00, \$44.00, \$4.44 or \$0.44?

FF. Suppose the following characters and their friends came to lunch in your school cafeteria. What would the cost of a regular lunch be for these groups?

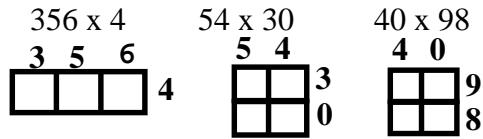
- Paul Bunyan and Babe
- Snow White and the Seven Dwarfs
- The Three Little Pigs
- Ramona and four friends
- Charlotte and nine baby spiders
- Lon PoPo, her two sisters and mom

See “note” in 1.03a) -EE.

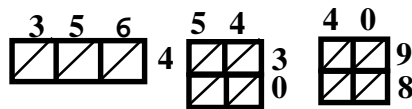
GG. Use lattice multiplication to practice basic facts and introduce multi-digit multiplication. Napier's Bones - Blackline Masters I - 55 and I - 56.

Notes and textbook references

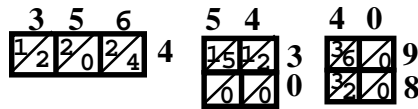
Step 1: Place numbers outside a grid as illustrated.



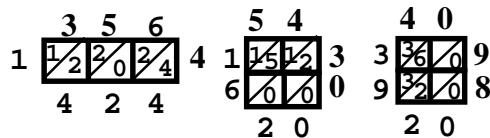
Step 2: Draw the diagonals as illustrated



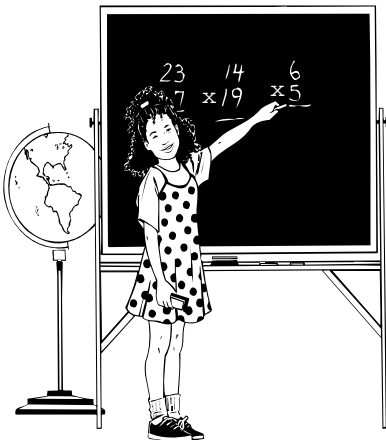
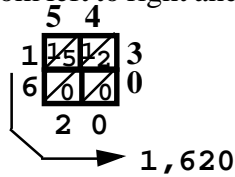
Step 3: Multiply (basic facts) putting ones digits of products in lower part of box and tens digits in upper part



Step 4: Add the numbers along the diagonals.



Step 5: Read product from left to right and around the corner.



b.) Estimation of products and quotients in appropriate situations.

A. Bring in recipes and have students figure out ingredient totals to feed the whole class. Using store ads have the students estimate how much money it would cost to make each recipe.

B. Have students compute the amount of gallons of gasoline needed to take a field trip to a location you specify. Extend this activity to having the students compute mileage, gas, food, and any entrance fees for a field trip of their choice. Then have them write a proposal to the principal detailing all information they have computed. Have them include an estimate of how much money each child would have to pay.

C. Give each student an individual bag of snacks such as mini-cookies or fruit snacks, and have the class estimate the number of cookies or fruit snacks that can be found in the entire class. (Have the class make the connection that each individual bag does not contain exactly the same amount or shape)

c.) *Relationships between operations.*

Notes and textbook references


A. Each student needs a collection of beans, cubes, or other small manipulatives and construction paper circles. Ask students to count out a specified number of beans. Have students share these equally among a designated number of circles. For example, 19 beans divided equally onto three circles means each circle has six beans and one is left over. Talk about the related multiplication and record both the division and the multiplication.

$$19 \div 3 = 6 \text{ R}1 \text{ and } 3 \times 6 = 18 + 1 = 19$$


B. Play **Huddle**, an outside game. The leader calls out a number between 1 and 10. Students rush to huddle in groups of that many. “Left-over” students go to the leader to see whether more groups can be made. Use the groups to practice counting by that number and to add repeatedly. Relate to multiplication.


C. Multiplication Trains. Assign values to connecting cubes: red = 1, blue = 2, green = 3, orange = 4, brown = 5, black = 9. Divide students into groups and have each group build a multi-color train with an assigned value. Example: a 12 train could be three orange, or six blue, or four green cubes. Record multiplication facts and relate to repeated addition.

Extension: Relate to division and have students write number sentences.

 = 5

 = 30

 = 2

 = 6

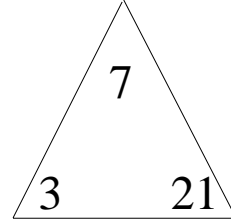
Notes and textbook references

Allow students to make triangular flash cards for their harder facts.

D. Show students a triangular card with three numbers corresponding to a multiplication fact. Using the card, have the students write two multiplication facts and two division facts based on the numbers given.

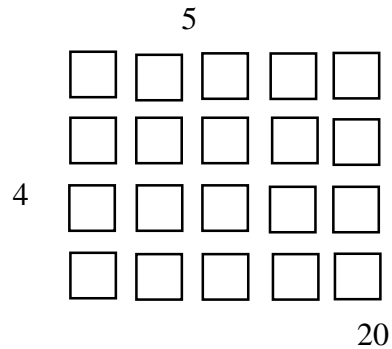
$$7 \times 3 = 21 \quad 21 \div 3 = 7$$

$$3 \times 7 = 21 \quad 21 \div 7 = 3$$



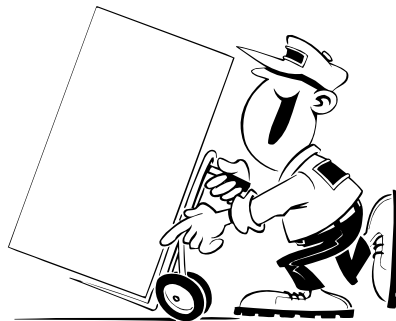
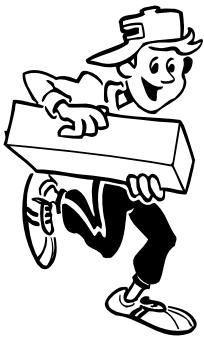
E. Provide graph paper (Blackline Master III - 6) or tiles for students to make rectangles whose sides are the factors in multiplication. Have students write the results as illustrated. Note the relationship to the way division problems are written.

$$\begin{array}{r} 5 \\ 4 \overline{)20} \end{array}$$

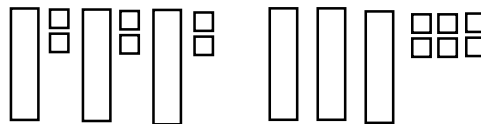


1.04 Use basic properties (identity, commutative, associative, order of operations) for addition, subtraction, multiplication, and division.

A. Prepare connecting cube towers of ones, twos, threes, and fours. These will be used to model the associative property of multiplication. $(2 \times 4) \times 3 = 2 \times (4 \times 3)$. Have students model two sets of four, using towers. Next, following the algorithm, students need to show two sets of four three times. Students need to count and recount $(2 \times 4) \times 3 = 24$. Now, ask students to model four sets of 3 towers two times and record. $[2 \times (4 \times 3) = 24]$. Does grouping the factors in different ways affect the product? Let students try other examples such as $3 \times (1 \times 2) = (3 \times 1) \times 2$ or $(2 \times 2) \times 3 = 2 \times (2 \times 3)$.



B. Use base 10 blocks to model 3 times different numbers. Have students explore different ways to group the blocks. For example, 3×12 may be modeled as $(3 \times 10) + (3 \times 2)$ or $3 \times 5 + 3 \times 7$.



Ask students to model and discuss alternate groupings for each example, writing the appropriate expressions, for these situations:

3×14

3×37

3×123

3×41

3×115 .

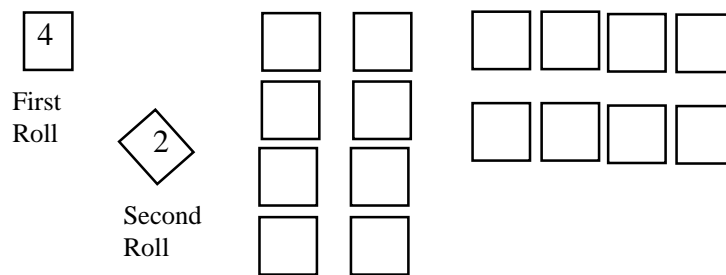
C. Use blocks, tiles, or paper squares to model expressions such as these:

$$(2 \times 3) + (2 \times 2) = 2 \times (3 + 2) \quad \text{and} \quad 6 \times 1 = 1 \times 6$$

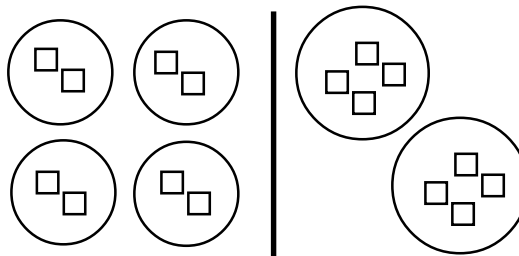
$$(3 \times 4) \times 2 = 3 \times (4 \times 2) \quad \text{and} \quad 48 \times 1 = 1 \times 48$$

After you model on the overhead, have students complete similar problems with manipulatives at their desks.

D. With color tiles on a piece of overhead film, let students take turns rolling a die twice and building the corresponding array, labeling the sides. Next, rotate the paper one quarter turn to model



the commutative property. Have the students also model this by first making 4 sets of two then reorganizing the game cubes to be 2 sets of four.



1.05 Use area or region models and set models of fractions to explore part-whole relationships.

Notes and textbook references

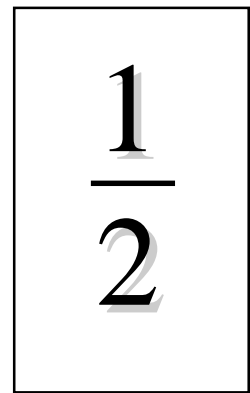
a.) Represent fractions concretely and symbolically (halves, fourths, thirds, sixths, eighths).

A. Make many different sets of fraction models.

1. The easiest are made from rectangles 2" x 24" cut from construction paper. With the students, fold, mark divisions, and label fractional parts. Each student should have at least a whole, halves, fourths, thirds, sixths, and eighths

Extension: Challenge students to figure out how to make models for fifths and tenths.

2. Using cupcake paper have students flatten to form circle. Fold in half for halves, fold again for fourths, and so on. After folding, students may color sections with markers and label each section.
3. Use paper plates for a larger group or class models. Divide each plate into fractions to be displayed.



B. Use connecting cubes to make **Fruity Fraction Bars**.

Model for students by making a fruity fraction bar of 8 connecting cubes containing 2 flavors (colors). Have students describe the bar. For example, the bar has 8 pieces in all; 4 pieces are cherry (red) so half is cherry; half is not cherry since half of 8 is 4. Repeat the activity using fruity fraction bars with different numbers of pieces of two colors (for example, 3 cherry and 5 lime). Later add more flavors, such as 12 pieces with 4 cherry, 2 blueberry, and 6 lemon.

Blackline Masters I - 36 through I - 40 provide fraction bars for many fraction activities.

C. Use pattern blocks to explore fractions. For example, if a yellow hexagon is a whole, which block is half? (A red trapezoid) Which block is $\frac{1}{3}$? (A blue parallelogram) What would the green triangle be called? If a hexagon is a whole, what would the value of 5 trapezoids be? Have the children model these fractions and others.

D. Use Cuisenaire™ rods to explore fractions. Find all the possibilities to make halves, thirds and fourths. (For example, yellow is half of orange. White is half of red.)

E. Divide a geoboard into halves, fourths, eighths. Transfer the divisions to dot paper (Blackline Master I - 41), and have the students shade several possibilities for each fraction. How many ways could you show one-fourth?

F. Divide children into groups of three (or four). Give each group 5 cookies cut from construction paper. Have them discuss how to share the cookies evenly and record each person's share. Continue by changing the number of cookies for each group. Share evenly and record.

G. Make fraction riddles with words. For example: Tell students to write the first....

first $\frac{2}{3}$ of "who" and first $\frac{2}{6}$ of "attack" **What**

$\frac{1}{2}$ of "half" and $\frac{1}{2}$ of "so" **has**

$\frac{4}{6}$ of "fourth" **four**

$\frac{3}{4}$ of "when" and $\frac{3}{4}$ of "else" **wheels**

$\frac{3}{3}$ of "and" **and**

$\frac{3}{4}$ of "flip" and $\frac{2}{6}$ of "escape" **flies?**

(A garbage truck)

Variations: Make the solution a riddle also. Or use this idea for names or vocabulary words. Have students create their own riddles.

H. Supply each student with two different colors of paper plates, 9 inches is a good size. Either draw a radius on each plate so students can cut on the line, or precut a radius before distributing. Students fit the two plates together by holding one plate at a right angle to the other and sliding the slits into each other, then flattening the plates. These two plates can be rotated to model fractions. For example, if the plates are white and blue, the plates can be rotated so that one-fourth of the circle is blue or white. Begin by asking students to model a variety of fractions, one-fourth, one-half, three-fourths, between one-fourth and one-half, etc. Spend a few minutes at the beginning of a class session providing opportunities for students to observe. Show some transparent markers of two colors on the overhead for about three seconds. There might be four blue markers and four orange markers. Ask students to use their paper plate models to show what fraction of the markers is blue. Repeat showing the overhead image for three seconds at a time until most students have successfully completed the task. Ask students to explain how they went about completing the task. Show a rectangle with parts shaded. Ask students to show what fraction was shaded. Whenever gathering data, for example when taking lunch counts, ask students to use their paper plate models to show what fraction of the class is ordering baked potato for lunch. Show connecting cube trains, for example, three red, two blue, and one yellow. Ask students to show what fraction of the train is red, etc. Students could also come to the front of the class with a train or some other collection and ask these questions.

I. Have students work in small groups to find different ways to cut a given rectangle into equal sections. Part of their task should be to explain how they know the sections are congruent in each case and what the relationship is between the parts and the whole rectangle. A group of four might be given four construction paper rectangles measuring 4" by 8". They might chose to fold one of these rectangles in half, creating two pieces measuring 2" by 8" , and identify each piece as $\frac{1}{2}$ of the whole. They might fold it twice, creating four pieces measuring 2" by 4" each and identify each as one-fourth of the whole. They might fold a third rectangle into eight pieces measuring two square inches and identify each piece as one-eighth. They might choose to create 16 2" by 1" pieces with the fourth rectangle. Each group might have a different sized set of four rectangles. As groups share their results with the whole class, a wonderful opportunity will arise to discuss the issue of how fractional parts must relate to the whole. One half of a 4" by 8" rectangle will be a different size (area) from half of a 2" by 4" rectangle. You might use different size candy bars to model these concepts.

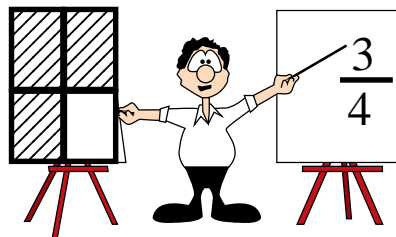
b.) Compare and order fractions (halves, fourths, thirds, sixths, eighths) using models and benchmark numbers (zero, one-half, one); describe comparisons.

A. Make several fraction sets with different-colored egg cartons. Leave white cartons uncut. Slice pink cartons to show halves, green cartons to show thirds, yellow cartons to show sixths, and blue cartons to show fourths. In groups have students compare the number of sections in the different models, always relating back to the whole carton.

B. Play the traditional game of **Battle** where the fraction closest to **one** wins. Use fraction bars or other models which make comparison of fractions easy. Note: this is not an appropriate activity at an abstract level for third graders. Through experiences and pictures they will learn that $\frac{2}{4}$ and $\frac{1}{2}$ and $\frac{3}{6}$ all shade the same amount of a region; however, they should not be expected to know that $\frac{2}{3}$ is greater than $\frac{5}{8}$ unless they have the models to compare. See Blackline Masters I - 36 through I - 40.

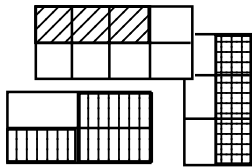
C. Use the rectangular fraction models made by the children with the fractional pieces cut apart to play a cover-up game. Make a fraction die for each group of three or four children or a spinner with appropriate fractions. See Blackline Masters I - 48 and I - 49. The object of the game is to be the first to completely cover two “whole” strips. If a student rolls $\frac{1}{2}$ and only needs $\frac{1}{4}$ to complete a strip, allow the child to trade the $\frac{1}{2}$ piece for two that are $\frac{1}{4}$ each.

Fraction kits made from six colors of construction paper help students learn equivalent fractions at concrete and pictorial levels.



D. Give each student an egg carton and different kinds of beans. Have students fill the egg carton with 2 different kinds of beans, one bean in each cup, so there is an equal number of each kind of bean. Discuss the total number of beans in the container, the number of each kind of bean, and the fractional part of the carton filled with each kind of bean (one-half). Repeat the activity using three kinds of beans to model thirds, four kinds to model fourths, and 6 kinds to model sixths.

E. Play **Fraction Three-In-A-Row**. Answer boards for students and the overhead master for the leader are located with the Blackline Masters I - 50 and I - 51. Children may choose any one of the six answer boards for a game, but the student may not change answer boards once a game has begun. The leader begins by covering all pictures on the master sheet. As a picture is uncovered, the student may mark the fraction if it is on the answer board chosen for the current game. To use an answer board more than once, give students markers to cover the fractions or have them mark with x's for the first round and completely blacken the square for the second time a board is used.

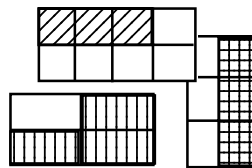


$\frac{4}{8}$	$\frac{5}{6}$	$\frac{1}{2}$
$\frac{1}{6}$	$\frac{3}{5}$	$\frac{2}{8}$
$\frac{2}{3}$	$\frac{6}{6}$	$\frac{1}{4}$

c.) Model and describe common equivalents, especially relationships among halves, fourths, and eighths, and thirds and sixths.

A. Each student will need six 24-inch strips of paper and a ruler. Leave one strip unfolded and label it as “one”. Students fold the first strip in half and estimate the length of each half. The children then measure, verifying that each half is 12 inches. Label each side of the strip as one-half ($\frac{1}{2}$). Using another strip, fold first into halves and then fold again to make fourths. Measure these sections. Label each section as one-fourth. Continue to make models for eighths, thirds, and sixths, labeling all sections. Note that children will need help in knowing how to fold their strips into thirds and then into sixths. When measuring these portions, students should focus on having each section be the same number of inches, rather than 24 inches divided into sixths will result in 4-inch sections. (Some students will see the relationships between fractions and divisions.) Strips may then be used to compare many different fractions.

B. Fraction Tic-Tac-Toe: Each student draws a tic-tac-toe board and writes a fraction using halves, thirds, fourths, sixths or twelfths as a denominator in each block. Teacher draws a fraction bar from bag and shows it to the group. (Use overhead fraction bars if playing with the entire class.) Discuss the name of the fraction. Each student who has this fraction will cover on game board with marker. First to get tic tac toe is winner. Variation: Students play with a partner. Take turns drawing a fraction bar. Decide on fraction represented. If either player has this fraction on tic-tac-toe board, it is covered. First to get tic tac toe is the winner.



$\frac{4}{8}$	$\frac{5}{6}$	$\frac{1}{2}$
$\frac{1}{6}$	$\frac{3}{5}$	$\frac{2}{8}$
$\frac{2}{3}$	$\frac{6}{6}$	$\frac{1}{4}$

C. Prepare a **Concentration** game matching fractions to pictures. Use the same picture cards as “flash cards” for children to identify mixed numbers or fractions. See Blackline Masters I - 42 through I - 47.

D. On the overhead put one blue chip and one red chip. Discuss $\frac{1}{2}$ of the group is blue and $\frac{1}{2}$ is red. Add one more blue and one more red. Now $\frac{2}{4}$ are blue and $\frac{2}{4}$ are red; but $\frac{1}{2}$ is still blue and $\frac{1}{2}$ is still red. Continue the pattern. In small groups have children use colored squares or connecting cubes to extend the activity and record the equivalent fractions on a chart. Use the same activity for other fractions. For example: a red chip, a yellow chip, a green chip, and a blue chip to model fourths.

E. Supply small groups of students with several colors of square inch tiles or connecting cubes. Ask students to build a “train” using several colors. Next, students color a representation of this train on grid paper. For example, students might build a train connecting 4 red tiles to 8 green tiles and then color a line on grid paper with 4 red squares connected to 8 green squares. After coloring the train on grid paper, students write what fraction of the train is red ($\frac{4}{12}$ or $\frac{1}{3}$) and what fraction is green ($\frac{8}{12}$ or $\frac{2}{3}$). This might be repeated with other collections of objects. Students might scoop a handful of two-colored counters, shake and drop. They could then draw a picture of the result and label the fractional parts. For example, a student might scoop up 15 counters and toss. If 7 yellow circles and 8 red ones, the yellow counters are $\frac{7}{15}$ and the red are $\frac{8}{15}$.

F. Ask students to search through discarded magazines for pictures of regions and objects that could represent fractions. Have them cut out the pictures, glue them onto background paper and write about the fractions shown. They might find pictures of pizzas with distinct sections of different toppings, or ice cream cones with different flavors. They might create their own by cutting out pictures of dogs and cats and labeling the fractional part represented by dogs or cats. Small groups of students might want to work together to create a “picture book” all about $\frac{1}{2}$ or some other fraction. These books could be shared with younger students.

G. Expand upon the idea of students creating a book all about one fraction. Emphasize that the book should picture many different names for the fraction. For example, the book about $\frac{1}{2}$ might have a page showing 5 cats and 5 dogs with a label stating $\frac{5}{10}$ are dogs and $\frac{5}{10}$ are cats. Another page might show an egg carton with 6 eggs and a label stating that $\frac{6}{12}$ of the eggs are missing.

H. Have students make their own fraction kits as described in the FAMILY MATH book published by EQUALS, Lawrence Hall of Science, University of California, Berkeley, California. Each student receives 5 different strips of construction paper measuring 3 inches by 18 inches. One strip is left whole, one is cut into two equal lengths and labeled $\frac{1}{2}$, a third is cut into 4 equal lengths and labeled $\frac{1}{4}$, a fourth is cut into 8 pieces and the fifth is cut into 16 pieces. You might decide that four pieces is enough and stop at $\frac{1}{8}$ pieces. Make labels for a wooden cube by writing the following fractions on circular labels: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{2}{8}$, and $\frac{2}{16}$. If you've decided not to work with $\frac{1}{16}$ pieces, replace the $\frac{1}{16}$ and $\frac{2}{16}$ labels with $\frac{2}{4}$ and $\frac{2}{8}$. Attach the labels to the six faces of the cube. Students play games in small groups with their fraction kits and one of these cubes. They take turns rolling the die and placing the fractional parts indicated on the die on the top of their whole strip. The object is to be the first to completely and exactly cover the whole strip. If a student has covered all but $\frac{1}{16}$ of his or her strip, then $\frac{1}{16}$ must be rolled on the die. If something other than $\frac{1}{16}$ is rolled, that student "misses" that turn. The reverse of this game can be played by covering the whole with the two $\frac{1}{2}$ pieces. Students take turns rolling the die and removing what shows on the die. If $\frac{1}{4}$ is rolled first, the student needs to trade $\frac{1}{2}$ for $\frac{2}{4}$ in order to remove $\frac{1}{4}$. This requires trading equivalent pieces of fractions. Have students invent their own new games using their fraction kits. After a while, add new pieces such as thirds, sixths and twelfths. Have students use rulers to measure even lengths of 6 inches on their 18 inch strips for these fractions - thirds. After dividing a second strip into thirds, each third can be divided in half by folding end to end to create sixths. The twelfths can be created by folding the 6 inch lengths end to end twice. This will require making new dice or spinners to generate the fractions for playing games.

I. Put students in groups of different sizes. Have them create a picture of themselves in a group poster. The group then describes the picture with fractions. Example: half of us have long hair, one-fifth of us is wearing glasses, three-fourths of us is female. Other groups can try to guess which picture is being described.



d.) Understand that the fractional relationships that occur between zero and one also occur between every two consecutive whole numbers.

A. Make a student number line. Hang a clothes line that starts with zero and ends with one. Give students fraction cards and have them arrange themselves in order between zero and one. You might start with $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{3}$, $\frac{2}{3}$ and $\frac{3}{4}$. Repeat using mixed numbers and with different end-points.

B. Read the book Measuring Penny by Loreen Leedy to your students and create an assignment using measurements as the main character in the story does. Set parameters such as measurements must be to the nearest half or nearest fourth.

C. Pass out broken rulers and give students a list of things to measure to the nearest half inch; the nearest fourth inch.

D. Pass out a manipulative that measures only a half inch and tell students this is their standard unit. Give items to be measured in half inches and then show how the half inches can be applied to the ruler for a more complete measure.

e.) Understand and use mixed numbers and their equivalent fraction forms.

A. Write three mixed numbers on the board. Each student is to choose one mixed number and make a poster which models the choice. Students may use pictures from magazines, glue light objects to the display, or draw pictures.

B. Have students make pizzas with English muffins or canned biscuits. Use different toppings to divide their pizzas into sections to show fractions such as halves, thirds, and fourths. After the pizzas are baked, have some students cut their pizzas into halves, others into thirds, and others into fourths. Use students and their “creations” to model mixed numbers.

C. Students can use magazines and newspapers to find articles, advertisements, and pictures that show mixed numbers in a variety of contexts. A book of numbers could be compiled to illustrate a variety of mixed numbers with Different “wholes” but the same fractional part .i.e. the HALF BOOK could have examples of $1 \frac{1}{2}$, $2 \frac{1}{2}$, $3 \frac{1}{2}$, etc.

D. Be sure that students encounter problems that require the ordering and computation of a variety of mixed numbers in real-world contexts. Exercises and problems from the textbook can be revised to provide lots of opportunity for practice with these numbers which can seem unwieldy for third graders.



two and a half skaters?



one and a half snowmobilers?

1.06 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

Notes and textbook references

A. Estimate the number of connecting cubes in a jar by grouping those visible by tens. After students guess, remove groups of 10, one set at a time and allow students to change their answers. Discuss why many had to change their estimates to a higher or lower number.

B. Give students a book with more than 100 pages. Have students estimate how many “groups of 10 pages” will be in the book. Have children go through the book and paper clip every ten pages together. Count the number of groups of ten. Write the total number of pages in standard form.

C. Have an estimation jar and constantly change contents. Have students estimate contents to closest 10 or 100 rather than exact number. Show them a set of 10 items from the jar to give a reference point.

D. Fill ziplock or brown paper bags with small “countable” items such as kidney beans, small rocks, paper clips, toothpicks, macaroni, seeds, small seashells, small pine cones, etc. Divide the class into groups. Have the group choose a bag to estimate, count, and group into 100’s, 10’s and 1’s.

E. Ask students to put one connecting cube in each pocket. Then ask the class to estimate how many pockets they think are in the class. Who has more pockets, boys or girls? Gather the cubes, making tens as they are collected. How many pockets did the girls have? the boys? the class?

By allowing children to work through and share their thinking and problem-solving strategies with the class, you will gain insights and diagnostic information about the students. It is important during such sessions that the teacher allow the students to decide which answers are correct if the answers differ.

*The questions from the
EQUALS book
Assessment
Alternatives in
Mathematics on pp.
40, 41 are helpful as
you work with
students.*

F. Establish an ongoing estimation center or bulletin board. Place a collection of objects (pencils, paper clips, beans, centimeter cubes, etc.) in some kind of container such as a jar or baggy. Baggies™ are convenient for tacking to bulletin boards. Include an envelope with blank papers, an envelope for depositing estimates, and a class list for students to check off their names as they deposit an estimate. Discuss estimating numbers in multiples of tens and/or hundreds and what kind of range would be appropriate for the collection of objects. Remember that this is an estimating task, not a “guess the right number of things” task. Any student within the agreed-upon range should be given credit. For example, if 43 paper clips are placed in a baggy, and the class decides that an estimate within 15 of the actual number is reasonable, then a student estimating 30, 40, or 50 paper clips should be credited for making a reasonable estimate. After everyone has had an opportunity to estimate, a team of students reads all the estimates and counts the number of objects in the container. Anyone with a reasonable estimate could be recognized on the class list with a sticker or highlight. The students doing the checking might set up the next estimating task and choose the next team to do the checking. The same objects could be used several times with different sized collections. For example, if 43 paper clips are presented first in a baggy, then the next task might be a collection of paper clips in a baggy twice as big, or three times as big. This provides an opportunity for students to base estimates upon previously-gathered information, which is a good estimating strategy. Be sure to leave the first collection of paper clips visible to encourage students to use this strategy.

G. Calculator Race. Have students work in pairs. One will use a calculator, the other will use mental math to answer basic math facts. After working 10 to 15 problems, have students discuss which was faster, the calculator or mental math. Discuss appropriate times for use of each method.

H. Give children answers and ask them to make up possible variations. “If the answer is 8, what could the question be?” “If the answer is 12, what could the question be?”

I. When students begin a new concept in mathematics, have them keep a log of the things they learned. Beginning multiplication and division are major third grade focuses.

J. Have the students list ways they have used math in their lives in the last week. Let all children list their individual uses; then, have a group of students make a bulletin board, deciding the classifications for the uses and displaying all examples.

K. Choose two grocery items that are the same type of product. Compare the weight/volume and the price to see which is the better buy, (i.e. unit pricing). Calculators are a must since students have not learned the necessary computations but may understand the unit cost. Use newspapers for actual pictures and ideas or get information during trips to the grocery store.

L. Find total class height: Pair students and have them measure each other and record in inches. Add all the heights together to find the “total class height”. Ask students to figure out how to convert the total inches into feet and inches. Gather the same information from other classes in your school. How does your total class height compare with the totals for other classes?

M. Have students find their arm span and the length of one of their arms. Given this information, have them determine their body width and discuss how they arrived at their answers.

N. Use sales pages from the newspaper and/or catalogs to research the cost of different items. Children record the prices they find to discover best buys. Compute the money saved from the most to the least expensive. Extension: What are the hidden costs for these items? If ordering from a catalog, is there a shipping charge? Where are the stores that have the best buys? Will you need to travel a long distance to get there? If so, what is your hidden cost? Show children how to use tax tables to figure taxes due and then add the cost of the items.

O. Make a table of contents for a book about Space. Decide how many pages your book will contain, the name of each chapter, and how many pages will be in each chapter. Chapters should begin on right-hand pages. How will they be numbered?

P. Give each student a calendar or hundred board (Blackline Master I - 2). Have the students connect numbers which are diagonally adjacent to form an X. Students add the two sets of two numbers connected by the diagonal lines. After they have worked several examples, ask students whether they can make some statements about what they expect the sums to be and why. In both calendar and hundred board examples, ask students to explain why they think the sums are equal. (Relate to patterns and relationship activities in GOAL 5).

Q. Give a target range (e.g. 125 - 150) that students are trying to reach as a sum. Ask students to enter a number into their calculators (for example, 47) and add another number with the calculator to reach the target. Have them record their efforts and try to adjust the second number to reach the sum. At a later time, reverse this activity by giving a number (for example, 56) and giving the target range for the difference (for example, 15 - 50) to emphasize subtraction.

R. Word Values. Students practice addition and multiplication with their calculators by finding the value of words. If consonants are worth 3 points and vowels are worth 5 points, what is the value of weather? (27). Use vocabulary related to current studies. Variations: Vary value of consonants and vowels to relate to multiplication facts being studied. (See “Valuable Words” chart , Blackline Master I - 52.)

S. Experiment, then extend class discoveries. Use calculators to allow students to extend information from in-school explorations. How much water runs from the water fountain in one minute? Using this information how much runs out in one hour? One day? How many steps from your classroom door to the cafeteria? How many times would you walk this distance to make a mile (approximately 5,280 steps)? How many miles will you walk going to the cafeteria this year?

T. Have students design a math game for their friends. Have children discuss the guidelines they need to establish. Be certain that students are able to identify the mathematical skills needed to play the game successfully. Before they make the “finished product,” have them play the game to see whether rules are clear and game is playable. See Blackline Master I - 53.

U. Every student needs a toy vehicle (brought in by students, teacher, or obtained from other sources). The students are to design and construct garages for their vehicles. Students should first make a plan that shows the steps they will use and then share that plan with others (a small group, whole class). Finally, have students construct their garages out of old file folders, index cards, etc. They may wish to color before assembling their display.

V. I have some pennies, nickels, and dimes. I put two coins in my hand. How much money could I have? Make a chart or list. Have students create other similar challenges. For example: Can you make change for a quarter using nine coins?

W. Your challenge is to make a schedule for the new soccer teams. There are three teams that wish to practice four days a week for 4 hours each. Each team wants two morning and two afternoon practices of at least two hours but not at the same time. The field is closed on Wednesdays. Variation: Make a schedule for the school day; a schedule for your family for the week-end.

X. A boy buys a skateboard for \$40.00, sells it for \$50.00, buys it back for \$60.00 and sells it for \$70.00. Does the student make or lose money? Use resources such as the Marilyn Burns book, *I Hate Mathematics*, to find other unusual problems.

Y. Pose problems like: I saw nine students riding dirt bikes and three-wheelers pass my house. I counted 23 wheels. How many students were riding three-wheelers and how many were riding dirt bikes? Have students write similar problems that challenge thinking.

Z. Have students solve $326 + 248$ using the traditional algorithm. Ask them to search for alternative ways to solve the problem. Encourage students to explore or write the steps in solving this problem in a logical order and to use manipulatives to model. For example, 326 is $325 + 1$ and 248 is $250 - 2$. I will add 325 and 250 because I can do that in my head. The answer is 575. I need to add 1 ($325 + 1$) and that is 576. Now I need to subtract 2 ($250 - 2$) and the answer is 574. Another way is to add $300 + 200$ is 500; $40 + 20$ is 60, so that is 560; $6 + 8$ is 14 and $560 + 14$ is 574.

AA. Generate **Broken Calculator** problems. For example, if the 5 and 9 keys on your calculator are broken, how can you use it to solve the problem $469 - 253$? How can $290 + 57$ be worked on this same calculator? Is there more than one way to do this? How can you do this with the fewest number of key strokes? Ask students to discuss these problems and create new ones.

BB. Use the calculator to write words by pressing the keys and turning the calculator upside down. After discovering which digits display certain letters, have children create problems whose solution will spell a word. Begin by giving them some riddles to solve. (1) Before Mary gave seven girls 2 cookies each, what did she say to them? (2) Ten more than 7,728 is which noisemaker? You may wish to have children work with partners. (Notice that to have an “o” at the end of a word, you must use the decimal form. For example, $0.351 + 0.453$ is “hobo” but $351 + 453$ is “hob”). Some other good riddles might be: Who might you find riding on a freight train? And what kind of goblin is found at Halloween?

CC. Start a collection of non-routine problems. Put these on cards to add to your collection each year. Allow students to choose a card and work with a partner to solve these “stumpers” in their free time. Here is one example: Eight children went to a birthday party. Each person rode a bicycle or a tricycle. If there were 19 wheels altogether, how many bicycles and how many tricycles were ridden to the birthday party? Allow children to work in teams and share ways they solved the problem since talking about alternate solutions helps children to understand different strategies.

DD. Working in pairs, have students circle 3 consecutive numbers on a hundred board. Tell the children to find the sum on the calculator and record. Next, divide that sum by 3 because they added 3 numbers. After several trials, have students talk to their partners to see whether they can make any conclusive statements. Allow children to continue to explore. Would this work for 3 numbers vertically or on a diagonal? For students who find a pattern, suggest that they repeat the procedures with 5 numbers, dividing the sum by 5. Can they predict what will happen with a sequence of 7 numbers? of 9 numbers? Would this work with 4 numbers? Why or why not?

EE. Use three containers with lids such as margarine containers. In one container put 2 green cubes and on the lid put 2 green dots. In a second container, put 2 red cubes and on the lid put 2 red dots. In the third container put one red and one green cube. On the lid put 1 green dot and 1 red dot. Next put the lids on the wrong containers. Tell students you have three containers and describe their contents. Show students lids on containers. Tell students to open only one container and pull out one block and predict what is inside the other containers. Have children experiment and discuss their reasoning and conclusions. Variation: Use boxes and construction paper squares.

FF. Bag foods that students can eat in snack-size ziplock™s. These might include popcorn, raisins, nuts, crackers, cereal and so on. Give each small group of students one of the bags. Have the group estimate how many pieces are in the bag and then share the snack equally. Discuss how to handle leftovers. Have students write number sentences indicating the outcome. For example, a group of four students estimated that there were 30 goldfish crackers in their bag. After dividing the crackers equally, each had 5 crackers and there were three left over. They wrote $4 \times 5 = 20$ and $20 + 3 = 23$ crackers. They also discussed their estimate, agreed that 25 would have been closer, and concluded that they made an acceptable estimate. Have students choose a money value for each snack and then determine how much their snack would cost. For example, the group of students with fish crackers decided that each cracker was worth 6¢. This same activity could be repeated with class collections. Prior to using a concrete tool, such as color tiles, ask students to share their collection equally among members of their working group. Then students can record the result by writing equations.

GG. Have students investigate telephone area codes using the digits 0 - 9. What are the original or classic area codes in North Carolina and the new codes introduced because of number overload in some cities or states. Old (pre-2000) phone books will list early codes. Determine how many area codes are possible. Remember 0 and 1 cannot start a code!



ASKING QUESTIONS

Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computation are a dime a dozen, but a good high-level open-ended question that gives students a chance to think is a treasure!

These questions might be used as teaching or “leading” questions as well as for assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. The questions and their responses will contribute a climate of thoughtful reflectiveness.

Some suggestions about assessment questioning:

- Prepare a list of possible questions ahead of time, but, unless the assessment is very formal, be flexible. You may learn more by asking additional or different questions.
- Use plenty of wait time; allow students to give thoughtful answers.
- For formal assessment, leading questions and feedback are not generally used, although some assessment techniques include teaching during the examination.
- Make a written record of our observations. A checklist may or may not be appropriate.

This is a starter list. You will want to build a collection of your own good questions.

Problem Comprehension

Can students understand, define, formulate, or explain the problem or task? Can they cope with poorly defined problems?

- What is this problem about? What can you tell me about it?
- How would you interpret that?
- Would you please explain that in your own words?
- What do you know about this part?
- Do you need to define or set limits for the problem?
- Is there something that can be eliminated or that is missing?
- What assumptions do you have to make?

Approaches and Strategies

Do students have an organized approach to the problem or task? How do they record? Do they use tools (manipulatives, diagrams, graphs, calculators, computers, etc.) appropriately?

- Where could you find the needed information?
- What have you tried? What steps did you take?
- What did not work?
- How did you organize the information? Do you have a record?
- Did you have a system? a strategy? a design?
- Have you tried (tables, trees, lists, diagrams...)?
- Would it help to draw a diagram or make a sketch?
- How would it look if you used these materials?
- How would you research that?

Relationships

Do students see relationships and recognize the central idea? Do they relate the problem to similar problems previously done?

- What is the relationship of this to that?
- What is the same? What is different?
- Is there a pattern?
- Let’s see whether we can break it down. What would the parts be?
- What if you moved this part?
- Can you write another problem related to this one?

Flexibility

Can students vary the approach if one is not working? Do they persist? Do they try something else?

- Have you tried making a guess?
- Would another recording method work as well or better?
- What else have you tried?
- Give me another related problem. Is there an easier problem?
- Is there another way to (draw, explain, say, ...) that?

Communication

Can students describe or depict the strategies they are using? Do they articulate their thought processes? Can they display or demonstrate the problem?

- Would you please reword that in simpler terms?
- Could you explain what you think you know right now?
- How would you explain this process to a younger child?
- Could you write an explanation for next year's students (or some other audience) of how to do this?
- Which words were most important? Why?

Curiosity and Hypothesis

Is there evidence of conjecturing, thinking ahead, checking back?

- Can you predict what will happen?
- What was your estimate or prediction?
- How do you feel about your answer?
- What do you think comes next?
- What else do you know?

Equality and Equity

Do all the students participate to the same degree? Is the quality of participation opportunities the same?

- Did you work together? In what way?
- Have you discussed this with your group? with others?
- Where would you go for help?
- How could you help another student without telling the answer?
- Did everybody get a fair chance to talk?

Solutions

Do students reach a result? Do they consider other possibilities?

- Is that the only possible answer?
- How would you check the steps you have taken, or your answer?
- Other than retracing your steps, how can you determine whether your answers are appropriate?
- Is there anything you have overlooked?
- Is the solution reasonable, considering the context?
- How did you know you were done?

Examining Results

Can students generalize and prove their answers? Do they connect the ideas to other similar problems or to the real world?

- What made you think that was what you should do?
- Is there a real-life situation where this could be used?
- Where else could this strategy be useful?
- What other problem does this seem to lead to?
- Is there a general rule?
- How were you sure your answer was right?
- How would your method work with other problems?
- What questions does this raise for you?

Mathematical Learning

Did the students use or learn some mathematics from the activity? Are there indications of a comprehensive curriculum?

- What were the mathematical ideas in this problem?
- What was one thing you learned (or two or more)?
- What are the variables in this problem? What stays constant?
- How many kinds of mathematics were used in this investigation?
- What is different about the mathematics in these two situations?
- Where would this problem fit on our mathematics chart?

Self-Assessment

Do students evaluate their own processing, actions, and progress?

- What do you need to do next?
- What are your strengths and weaknesses?
- What have you accomplished?
- Was your own group participation appropriate and helpful?
- What kind of problems are still difficult for you?

From *Assessment Alternatives in Mathematics*, a booklet from the California Mathematics Council and EQUALS

Patterns for Reviewing Addition Facts

In first and second grades children spend a significant amount of time learning addition and subtraction facts. However, many third graders still need time to study the facts. This presents an organized plan for review that builds on counting patterns. Notice that some facts fit into more than one category.

Begin with a number plus one which is the normal counting sequence. Write $0 + 1$, $1 + 1$, $2 + 1$ through $9 + 1$ in a column on the board. Ask children to tell you why they can give sums very rapidly. (Review a number less one also.)

Use counters to demonstrate that $4 + 1$ is commutative; that is, the addends may be arranged in any order ($4 + 1$ or $1 + 4$). Throughout your review of facts, continue to model for children that addition is commutative.

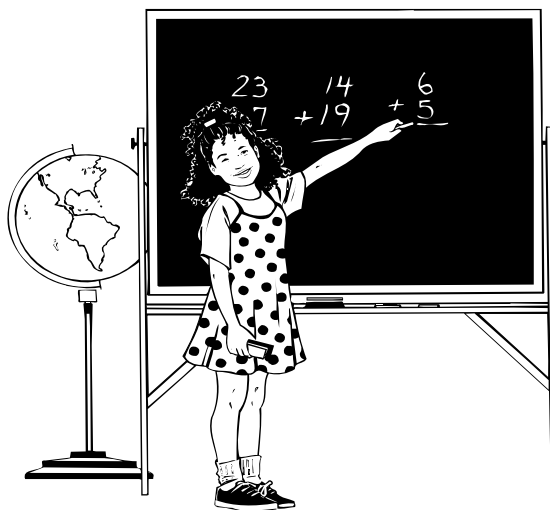
($a + b = b + a$) Be sure to use a variety of models, including the children themselves.

Plus 1 $0 + 1, 1 + 1, 2 + 1, 3 + 1, 4 + 1, 5 + 1, 6 + 1, 7 + 1, 8 + 1, 9 + 1$

The next review should be of **doubles**. Doubles are usually very easy for children. While “facts” are single-digit addends, children will learn doubles such as $10 + 10$ and $50 + 50$.

Doubles $0 + 0, 1 + 1, 2 + 2, 3 + 3, 4 + 4, 5 + 5, 6 + 6, 7 + 7, 8 + 8, 9 + 9$

When you begin with doubles plus one, teach the new facts by relating them to what children have already reviewed. Put up title (Doubles + 1) and one example of Doubles + 1 problem, i.e. $2 + 2 = 4$ and $2 + 3 = 5$. Show that $2 + 3$ is also $2 + (2 + 1)$. Use counters to model the examples as well as writing the facts. Have children discuss why $2 + 3 = 5$ is called Doubles + 1. Have children list other Doubles + 1's problems and answers. Children may notice all answers are odd. Why?



Doubles + 1 $0 + 1, 1 + 2, 2 + 3, 3 + 4, 4 + 5, 5 + 6, 6 + 7, 7 + 8, 8 + 9$

Before moving to the next classification of facts, be certain children have related subtraction to the addition facts you have reviewed. For example, **Sharing numbers** relates back to doubles. Sharing numbers $2 + 4 = 6$ reverts to $3 + 3 = 6$ if the 4 shares 1 with the 2. Children may notice that these combinations are also example of doubles + 2 (For example, $3 + 5$ is a double ($3 + 3$) + 2.). Note that all answers are even.

Sharing Numbers $0 + 2, 1 + 3, 2 + 4, 3 + 5, 4 + 6, 5 + 7, 6 + 8, 7 + 9$

Put up all nines ($9 + 1, 9 + 2, 9 + 3 \dots$) on the board and have the children provide answers. Ask children to look for a pattern. Children notice that the digit in the ones place in the sum is one less than the number added to nine. Other children will see that the answer is one less than 10 plus the number added to nine. Manipulatives will reinforce this use of 10 to simplify addition. These are both good mental strategies.

Nines $9 + 0, 9 + 1, 9 + 2, 9 + 3, 9 + 4, 9 + 5, 9 + 6, 9 + 7, 9 + 8$

Numbers plus zero frequently become difficult for children when they first are introduced to multiplication. Review these by modeling the facts.

Plus 0 $0 + 0, 0 + 1, 0 + 2, 0 + 3, 0 + 4, 0 + 5, 0 + 6, 0 + 7, 0 + 8, 0 + 9$

The following are the most difficult facts for most children. Encourage them to develop their own strategies for remembering them. If children count on their fingers, do not make an issue of this; but, help them to develop other methods (for example, counting on). Facts which include having children talk about how they remember harder facts may help their classmates.

Others $6 + 2, 7 + 2, 8 + 2, 3 + 6, 3 + 7, 3 + 8, 4 + 7, 4 + 8, 5 + 8$

Remember number facts all year as an on-going project and use games as motivation. By the end of third grade it is desirable for children to have worked with addition and subtraction facts so frequently that they are memorized.



Tips For Parents

1. Keep drill sessions short and interesting.
2. Express confidence in child's ability to memorize facts.
3. Practice daily but vary drills.
4. Praise efforts and record progress.
5. Memorize a few new facts at a time and review constantly.